Math 110 Algebra - Fall 2018

Exponents

After completing this section, students should be able to:

- Explain the product rule, quotient rule, and power rule for exponentiation by writing out powers of a variable as repeated multiplication.
- Rewrite expressions with negative exponents so that all exponents are positive.
- Use the product rule, quotient rule, and power rule to simplify exponential expressions.
- Decide whether equations involving exponents are always true or not by testing examples.
- Rewrite fractional exponents in terms of radicals.
- Calcuate numbers raised to fractional exponents without a calculator.

Here are the exponent rules:

1. The Product Rule

2. The Quotient Rule

3. The Power Rule

4. Power of Zero

5. Negative Exponents

6. Fractional Exponents

7. Distribute an Exponent over a Product

8. Distribute an Exponent over a Quotient

Example. Simplify the following expressions and write your answers without negative exponents:

(a) $\dfrac{3x^{-2}}{x^4}$

(b) $\dfrac{4y^3}{y^{-5}}$

Example. Simplify the expression and write your answer without negative exponents:

$$\frac{y^3z^5}{7z^{-2}y^7}$$

Example. Simplify the expression and write your answer without negative exponents:

$$\left(\frac{25x^4y^{-5}}{x^{-6}y^3}\right)^{3/2}$$

END OF VIDEOS

Question. Which statements are true for all positive values of x and y? (Select all that apply.)

A. $x^2 + y^2 = (x + y)^2$

B. $x^2y^2 = (xy)^2$

Question. Which expression is equivalent to $\dfrac{y^4}{4x^{-2}}$?

A. $4x^2y^4$

B. $\dfrac{x^2y^4}{4}$

C. $\dfrac{y^4}{4\sqrt{x}}$

D. $\dfrac{\sqrt{x}y^4}{4}$

Question. Without using a calculator, decide which of the following expressions is equal to -3. (Select all correct answers.)

A. $-81^{1/4}$

B. $(-81)^{1/4}$

C. $-27^{1/3}$

D. $(-27)^{1/3}$

Example. Simplify and write your answer without negative exponents.

$$\left(\frac{49x^8y^{-5}}{x^{-6}y^3}\right) \qquad \left(\frac{49x^8y^{-5}}{x^{-6}y^3}\right)^{3/2}$$

Extra Example. Simplify and write your answer without negative exponents.

$$\left(\frac{16a^{-2}b^{8}c^{20}}{a^{6}b^{-4}}\right)^{3/4}$$

Radicals

After completing this section, students should be able to:

1. Determine whether equations involving radicals are always true or not by testing with examples.
2. Simplify expressions involving radicals by ”pulling as much as possible” out of the radical sign.
3. Rationalize the denominator by multiplying the numerator and denominator by a radical expression or by a conjugate of a radical expression.

Rules for Radicals

- $\sqrt[n]{a \cdot b} =$

- $\sqrt[n]{\dfrac{a}{b}} =$

- $a^{m/n} =$

Example. Compute $(25)^{-3/2}$

Example. Simplify. (Assume all variables represent positive numbers.)

$$\sqrt{60x^2y^6z^{-11}}$$

Example. Rationalize the denominator. (Assume x represents a positive number.)

$$\frac{3x}{\sqrt{x}}$$

END OF VIDEO

Review. Which formulas are correct? (Select all that apply.)

A. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

B. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

C. $\sqrt{a-b} = \sqrt{a} - \sqrt{b}$

D. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

E. $\sqrt{a^2+b^2} = a+b$

Question. Without using a calculator, compute $16^{-3/4}$.

A. 12

B. -12

C. $\frac{1}{12}$

D. 8

E. -8

F. $\frac{1}{8}$

Example. Simplify the radical.

$$\sqrt{24x^6y^5}$$

$$\sqrt{\frac{x^9y^{16}}{64z}}$$

Example. Simplify.

$\sqrt{5xy^7}\,\sqrt{15x^3y^3}$

Question. Do these two expressions represent the same thing or different things? (Assume x and y represent positive numbers.)

A. $\dfrac{3\sqrt{x}}{5\sqrt{y}}$

B. $\dfrac{3\sqrt{xy}}{5y}$

Question. Do these two expressions represent the same thing or different things? (Assume a and b represent positive numbers.)

1. $\dfrac{5+\sqrt{a}}{3-\sqrt{a}}$
2. $\dfrac{15+8\sqrt{a}+a}{9-a}$

Extra Example. Rewrite by rationalizing the denominator:

$$\frac{\sqrt{a}+7}{\sqrt{a}+1}$$

Extra Example. Rewrite by rationalizing the denominator:

$$\frac{\sqrt{4x} + \sqrt{y}}{2\sqrt{3xy}}$$

Question. True or False: $\sqrt{x^2 + 9} = x + 3$

Factoring

After completing this section, students should be able to:

- Factor an expression by pulling out common factors from each term.
- Factor a difference of two squares.
- Factor a sum or difference of two cubes.
- Factor some expressions with four terms by grouping.
- Factor quadratics when possible.
- Combine these factoring methods to factor a complex expression several times.

1. **Pull out the greatest common factor**
   Example. Factor $15 + 25x$

   Example. Factor $x^2y + y^2x^3$

2. **Factor by grouping**

Example. Factor $x^3 + 3x^2 + 4x + 12$

3. **Factor quadratics**

Example. Factor $x^2 - 6x + 8$

Example. Factor $10x^2 + 11x - 6$

4. **Difference of squares**

$a^2 - b^2$

Example. Factor $x^2 - 16$

Example. Factor $9p^2 - 1$

5. **Difference or sum of cubes**

$a^3 - b^3 =$

$a^3 + b^3 =$

Example. Factor $y^3 + 27$

END OF VIDEO

Question. Which of these expressions DOES NOT factor?

A. $x^2 + x$

NEXT TIME MAKE THIS ONE HARDER LIKE $9x^2 - 25$

B. $x^2 - 25$

C. $x^2 + 4$

D. $x^3 + 2x^2 + 3x + 6$

E. $5x^2 - 14x + 8$

Question. Factoring by grouping is handy for factoring which of these expressions? How can you tell?

A. $10u^8v^6 + 15u^2v^2x^3$

B. $2y^5 - 7y^4 - 14y + 49$

C. $2z^2 + 3z - 14$

Question. What are some of the main techniques of factoring?

Extra Example. Factor $2z^2 + 3z - 14$.

Extra Example. Factor $-5v^2 - 45v + 50$

Rational Expressions

After completing this section, students should be able to:

- Simplify rational expressions by factoring and cancelling common factors.
- Find the least common denominator for two or more rational expressions.
- Add and subtract rational expressions.
- Multiply and divide rational expressions.
- Simplify complex rational expressions that involve sums or differences of rational expressions divided by sums or differences of rational expressions.
- Recoginze common errors in simplifying rational expressions such as cancelling across a plus or minus sign.

Reduce to lowest terms

Example. Simplify $\dfrac{21}{45}$ by reducing to lowest terms.

Example. Simplify $\dfrac{3x+6}{x^2+4x+4}$ by reducing to lowest terms.

Multiplying and Dividing

Example. Compute

a) $\frac{4}{3} \cdot \frac{2}{5}$

b) $\dfrac{\frac{4}{5}}{\frac{2}{3}}$

Example. Compute $\dfrac{\frac{x^2+x}{x+4}}{\frac{x+1}{x^2-16}}$

Adding and Subracting

Example. Subtract $\frac{7}{6} - \frac{4}{15}$

Example. Add $\frac{3}{2x+2} + \frac{5}{x^2-1}$

END OF VIDEO

Question. True or False:

1. $\dfrac{5xy}{5xz} = \dfrac{y}{z}$

2. $\dfrac{5x + y}{5x + z} = \dfrac{y}{z}$

Example. Subtract

$$\frac{x}{4x-9} - \frac{x+6}{5x}$$

Example. Compute

$$\frac{x}{9-2x}+\frac{6x}{2x-9}$$

Example. Simplify $\dfrac{\dfrac{x}{7}-\dfrac{7}{x}}{\dfrac{8}{7}-\dfrac{8}{x}}$

Extra Example. Subtract $\dfrac{2}{x^2 - 6x - 16} - \dfrac{1}{x^2 - 11x + 24}$

Extra Example. Simplify

$$\frac{\dfrac{4x-12}{x^2+2x}}{\dfrac{x^2-x-6}{x^2}}$$

Solving Quadratic Equations

After completing this section, students should be able to:

- Rewrite a quadratic equation in standard form
- Solve a quadratic equation by factoring
- Solve a quadratic equation using the quadratic formula
- Recognize when a quadratic equation has no real roots
- Solve a linear or quadratic equation in which the coefficients are themselves variables, to get solutions in terms of the other variables.

Example. Find all real solutions for the equation $y^2 = 18 - 7y$

Example. $w^2 = 121$

Example. Find all real solutions for the equation $x(x+2) = 7$

Example. Find all real solutions for the equation $\frac{1}{2}y^2 = \frac{1}{3}y - 2$

END OF VIDEO

Example. Find all real solutions for the equation $(x+2)^2 = 3x+7$

Example. Find all real solutions for the equation

$$x^4 - 8x^2 - 9 = 0$$

Example. Find all real solutions for the equation

$$3x^{2/3} + 5x^{1/3} = 8$$

Extra Example. Solve for r:

$$\frac{S}{h} - 2pr = pr^2$$

Extra Example. Solve for p:

$$\frac{S}{h} - 2pr = pr^2$$

Solving Rational Equations

By the end of this section, students should be able to:

- Solve rational equations by clearing the denominator
- Recognize and exclude extraneous solutions to rational equations
- Solve rational equations with several variables for one of the variables

Example. Solve: $\dfrac{x}{x+3} = 1 + \dfrac{1}{x}$

Example. Solve: $\dfrac{4c}{c-5} - \dfrac{1}{c+1} = \dfrac{3c^2+3}{c^2-4c-5}$

END OF VIDEO

Example. Solve $\dfrac{8x^2}{x^2-1} = \dfrac{4}{x-1} + \dfrac{9x}{x+1}$

Extra Example. Solve $4 - \dfrac{1}{v+1} = \dfrac{7}{v+4}$

Review. What are the main steps to solving a rational equation?

Question. What is an extraneous solution?

Solve $z = \dfrac{5P + c}{P}$ for P

Example. Solve for w: $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{w} = 6$

Extra Example. Solve $\frac{S}{h} - 2pr = pr^2$ for h.

Extra Example. Solve $\frac{S}{h} - 2pr = pr^2$ for p

Extra Example. Solve $\frac{S}{h} - 2pr = pr^2$ for r

Find the error:

Find the Errors:

$$x - \frac{20}{x} = 1$$

$$x\left(x - \frac{20}{x}\right) = 1$$

$$x^2 - 20 = 1$$

$$x^2 - 21 = 0$$

$$x^2 = 21$$

$$x = \pm\sqrt{21}$$

solutions are $x = \sqrt{21}$
$x = -\sqrt{21}$

$$x - \frac{20}{x} = 1$$

$$x \cdot \frac{x}{x} - \frac{20}{x} = 1$$

$$\frac{x^2}{x} - \frac{20}{x} = 1$$

$$\frac{x^2 - 20}{x} = 1$$

$$x - 20 = 1$$

$$x = 21$$

solution is $x = 21$

Radical Equations

By the end of this section, students should be able to

- Solve an equation with square root signs by isolating the square root and squaring both sides.
- Solve an equation with fractional exponents by isolating the fractional exponent and taking both sides to the reciprocal exponent power.
- Solve an equation with fractional exponents that can be written in quadratic form.
- Solve an equation with radical signs and several variables for one of the variables.

Example. Find all real solutions for the equation $x + \sqrt{x} = 12$

Example. Find all real solutions for the equation $2p^{4/5} = \dfrac{1}{8}$

END OF VIDEO

Question. Which step is wrong in this "solution"?

$$y + \sqrt{3y+1} = 3 \tag{0.1}$$
$$(y + \sqrt{3y+1})^2 = 3^2 \tag{0.2}$$
$$y^2 + (\sqrt{3y+1})^2 = 9 \tag{0.3}$$
$$y^2 + 3y + 1 = 9 \tag{0.4}$$
$$y^2 + 3y - 8 = 0 \tag{0.5}$$
$$y = \frac{-3 \pm \sqrt{9+32}}{2} \tag{0.6}$$

Question. What is the best first step to solve this equation? $y + \sqrt{3y+1} = 3$

A. Isolate y by subtracting $\sqrt{3y+1}$ to both sides.

B. Isolate $\sqrt{3y+1}$ by subtracting y from both sides.

C. Square both sides.

What are the solutions to $y + \sqrt{3y + 1} = 3$?

Example. What is a good next step to solve this equation for t?

$$(t + 3)^{2/3} = 4$$

A. Take the cube root of both sides.

B. Cube both sides.

C. Take the 2/3 power of both sides.

D. Take the 3/2 power of both sides.

Solve $(t + 3)^{2/3} = 4$.

Example. Solve for g: $T = 2\pi\sqrt{\dfrac{L}{g}}$

Extra Example. $5c^{2/5} - 11c^{1/5} + 2 = 0$

Extra Example. Find all real solutions for the equation $\sqrt[3]{5x^2 - 4x} - x = 0$

Absolute Value Equations

After completing this section, students should be able to

- Solve equations with absolute value signs and linear expressions inside the absolute value signs.
- Recognize when absolute value equations have no solutions.

Example. Solve the equation $3|x| + 2 = 4$

Example. Solve the equation $|3x + 2| = 4$

Example. Solve the equation $5|4p - 3| + 16 = 1$

END OF VIDEO

Example. Solve $3|1-2x|-1=8$

A. 1

B. -1

C. 1 or -1

D. -1 or 2

E. 1, -1, 2, or -2

F. No solution.

Example. Solve $5|x-3|+7=2$

A. 2

B. 2 or -2

C. 4

D. 4 or -4

E. 2 or 4

F. No solution.

Review for Test 1

Question. True or False: $\sqrt{x^2 + y^2} = x + y$

Consider the following types of equations and methods.

Type of Equation:

1. Rational equation
2. Quadratic equation or factor
3. Equation with fractional exponents
4. Radical equation
5. Absolute value equation

Method

1. Clear the denominators
2. Quadratic formula
3. Raise both sides to an exponent
4. Take a root of both sides
5. Rewrite absolute value as cases
6. Basic methods: distributing, moving all terms with the variable to one side and all other variables to the other side, factoring out the variable, and dividing

For each of the following equations, decide on the **type** of equation and the **method(s)** you would use to solve it. You do not actually have to solve the equation.

1. $\frac{3}{2x} = \frac{x}{4}$

2. $5x^5 = 17$

3. $|4 + x| = 6$

4. $3x(x + 2) = 5(x - 1)$

5. $A = P(1 + rt)$ for r

6. $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{c}$ for b

7. $y = \dfrac{4 - 3x}{5x - 9}$ for x

8. $y = 3 - \sqrt{4 + x}$ for x

Absolute Value Inequalities

After completing this section, students should be able to:

- Solve inequalaties involving absolute value signs with linear expressions inside the absolute value signs.
- Recognize when absolute value inequalities have no solutions.
- Recognize when absolute vale inequalities have all real numbers as solutions.

Example. What x-values satisfy $|x| < 5$?

Example. What x-values satisfy $|x| \geq 5$?

Example. Solve $|3 - 2t| < 4$

Example. Solve $|3 - 2t| > 4$

Example. Solve $2|4x + 5| + 7 \geq 1$

END OF VIDEO

Example. If we write $|4x + 1| \leq 3$, what does this mean?

A. x is less than or equal to 3

B. x is between -3 and 3, inclusive

C. $4x + 1$ is less than or equal to 3

D. $4x + 1$ is between -3 and 3, inclusive

Example. Solve $|4x + 1| \leq 3$

Example. If we write $|4x + 1| > 3$, what does this mean?

A. $4x + 1$ is bigger than 3

B. $4x + 1$ is between -3 and 3

C. $4x + 1$ is both less than -3 and bigger than 3

D. either $4x + 1$ is less than -3, or else $4x + 1$ is bigger than 3

Example. Solve $|4x + 1| > 3$

Question. Which inequaltiy has ”all real numbers” as its solution?

A. $|5x + 6| \leq -5$

B. $|5x + 6| \geq -5$

C. $|5x + 6| \leq 5$

D. $|5x + 6| \geq 5$

Compound Linear Inequalities

After completing this section, students should be able to:

- Solve compound linear inequalities connected by an ″AND″ statement or an ″OR″ statement, or written in the form $\square < \square < \square$
- Graph solutions of inequalities on a number line.
- Write solutions to inequalities in interval notation.
- Recognize when compound linear inequalities have no solutions or have all real numbers as solutions.

Example. Solve: $-5(x+2)+3>8$

Example. Solve: $3x - 4 \leq x - 8 \quad \text{OR} \quad 6x + 1 > 10$

Solve: $-\frac{2}{3}y > -12 \quad \text{AND} \quad -4y + 2 > 5$

Example. Solve: $-3 \leq 6x - 2 < 10$

END OF VIDEO

Example. Graph the solutions to $x \leq 5$ AND $x > 0$ on the number line. Then write the solution in interval notation.

Example. Graph the solutions to $x \leq 5$ OR $x > 0$ on the number line. Then write the solution in interval notation.

Example. Find the solutions to the system of inequalities $6 - 3y \geq 5y$ OR $2y \geq 4 + y$.

Question. Which system of inequalities has no solutions?

A. $x < 7$ AND $x > 2$

B. $x < 7$ OR $x > 2$

C. $x < 7$ AND $x > 10$

D. $x < 7$ OR $x > 10$

Example. Which of the following expressions are equivalent to: ″$x \leq 1$ OR $x > 3$″ (select all that apply)

A. $1 \geq x > 3$

B. $3 < x \leq 1$

C. $[1, 3)$

D. $(-\infty, 1] \cup (3, \infty)$

E. $x \leq 1$ AND $x > 3$

Extra Example. Find the solutions to the system of inequalities $2 - 4x < 5$ AND $2x + 5 \leq 13 - x$.

Polynomial and Rational Inequalities

After completing this section, students should be able to:

- Solve a polynomial or rational inequality by making a sign chart.
- Determine whether the endpoints of intervals should be included or not included in the solution of a polynomial or rational inequality and explain why.
- Graph the solution to a polynomial or rational inequality on the number line.
- Write the solution to a polynomial or rational inequality using interval notation.
- Explain why it is not correct to solve the inequality $x^2 < 4$ by taking the square root of both sides to get $x < 2$.

Example. Solve $x^2 < 4$

Example. Solve $x^3 \geq 5x^2 + 6x$

Example. Solve $\dfrac{x^2+6x+9}{x-1} \leq 0$

END OF VIDEO

Example. The solution to $(x+2)(x+1) \leq 0$ is

A. $(-\infty, -1]$

B. $(-\infty, -2]$

C. $[-2, \infty)$

D. $[-2, -1]$

E. $(-\infty, -2] \cup [-1, \infty)$

Example. The solution to $\dfrac{x+3}{x+1} > 0$ is

A. $(-1, \infty)$

B. $(-3, \infty)$

C. $(-\infty, -3) \cup (-1, \infty)$

D. $(-3, -1)$

What about $\dfrac{x+3}{x+1} > 0$?

Example. Solve $x^2 + x + 1 > 0$

True or False: The inequality $\dfrac{3}{x+1} \geq \dfrac{4}{6-x}$ has the exact same solutions as the inequality $3(6-x) \geq 4(x+1)$.

Example. Solve $\dfrac{3}{x+1} \geq \dfrac{4}{6-x}$

Distance Formula

After competing this section, students should be able to:

- Find the distance between two points on the plane.
- Explain how the distance formula is related to the Pythagorean Theorem for triangles.

Example. Find the distance between the points $P(-1, 5)$ and $Q(4, 2)$.

Midpoint Formula

After competing this section, students should be able to:

- Explain the midpoint formula in terms of average values.
- Find the midpoint of a line segment, given the coordinates of the endpoints.

Example. Find the midpoint of the segment between the points $P(-1,5)$ and $Q(4,2)$.

Circles

After competing this section, students should be able to:

- Find the equation for a circle given its center and radius.
- Find the center and radius of a circle, given its equation in standard form.
- Complete the square to rewrite the equation for a circle in standard form.
- Find the equation of a circle given two points at the end of a diameter, or given its center and one point on the circle.
- Explain how the equation for a circle is related to the distance formula or the Pythagorean Theorem for triangles.

Example. Find the equation of a circle of radius 5 centered at the point $(3, 2)$.

Note. The equation of a circle with radius r centered at the point (h, k) is given by:

Example. Does this equation represent a circle? If so, what is the center and what is the radius?

$$(x-5)^2+(y+6)^2=5$$

Example. Does this equation represent a circle? If so, what is the center and what is the radius?

$$9x^2 + 9y^2 + 72x - 18y + 36 = 0$$

END OF VIDEO

Example. What is the equation of a circle with center $(3, -2)$ and radius 5?

Example. Does this equation represent a circle? If so what is the center and what is the radius?

$$3x^2 + 3y^2 - 18x + 30y + 6 = 0$$

Example. Find the center and radius of this circle.

$$x^2 + y^2 + 14x - 18y - 2 = 0$$

Example. Find the equation of a circle whose diameter has endpoints $(-3, -6)$ and $(5, 4)$.

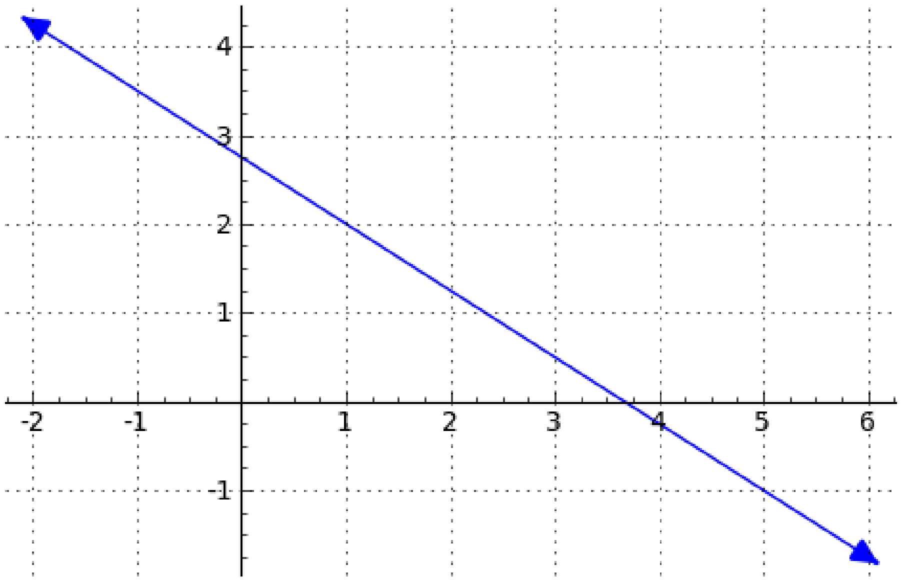
Extra Example. Given the point $P(1,5)$, find all points whose y-coordinate is 3 that are a distance of 7 units from point P.

Lines

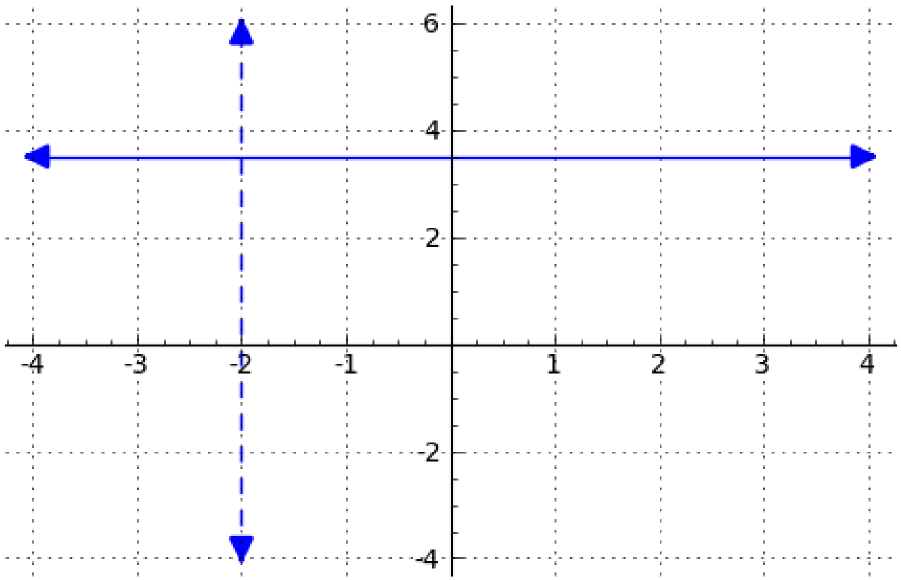
After finishing this section, students should be able to:

- Find the equation of a line given a slope and a y-intercept, or a slope and one point, or two points.
- Determine the equation of a line in an applied context, such as given a table of values for two variables, or given an initial value and a rate of change.
- Interpret the slope and y-intercept in the equation of a line in an applied context.
- Use the equation of a line to make predictions in an applied context.
- Find the equations of vertical and horizontal lines.

Example. Find the equation of this line.



Example. Find the equation of these lines.



Example. Find the equation of the line through the points $(1, 2)$ and $(4, -3)$.

END OF VIDEO

Example. A line has slope $\frac{1}{5}$ and passes through the point $P(3, 1)$. The equation for this line is $y = \frac{1}{5}x + \square$. Fill in the box.

Example. A company sells hats. If the selling price is \$8, they sell 300. If the selling price is \$6, they sell 420. The relationship between the selling price and the number sold is linear. Which pair of points can be used to construct the equation of this line? (Choose all that apply.)

A. $(8,6)$ and $(300,420)$

B. $(8,300)$ and $(6,420)$

C. $(8,420)$ and $(6,300)$

D. $(300,8)$ and $(420,6)$

How many hats would the company be able to give away if it offered them for free?

At what price would the company not be able to sell ANY hats?

Example. Write the equation for a line through the points $P(4, 2)$ and $Q(4, -1)$.

A. $x = 4$

B. $y = 4$

C. $x = 2$

D. $y = 2$

E. Undefined.

Example. Which of the following pieces of information is enough to find the equation for a line? (Select all that apply.)

A. the slope and the y-intercept

B. the slope and the x-intercept

C. the x-intercept and the y-intercept

D. the slope and a point on the line that is not an intercept

E. two points on the line

Extra Example. The deer population in Farrington village is growing at a constant rate of 60 deer per year. Two years after Mark McCombs moved there, the population was 340. What was the population 10 years after he moved in?

Extra Example. Snow is falling at a rate of $\frac{3}{4}$ inch per hour. How long will it take for the snow height to increase by 6 inches?

Parallel anad Perpendicular Lines

After completing this section, students should be able to:

- Determine if two lines are parallel, perpendicular, or neither based on their equations.
- Given an equation for a line and a point not on the line, find the equation of a line parallel to the given line that goes through a given point.
- Given an equation for a line and a point, find the equation of a line perpendicular to the given line that goes through a given point.

Example. Find the equation of a line that is parallel to the line $3y - 4x + 6 = 0$ and goes through the point $(-3, 2)$.

Example. Find the equation of a line that is perpendicular to the line $6x + 3y = 4$ and goes through the point $(4, 1)$.

Example. Find the equation of the line that is parallel to $y = 3$ and goes through the point $(-2, 1)$.

Example. Find the equation of the line that is perpendicular to $y = 4$ and goes through the point $(3, 4)$.

END OF VIDEO

Example. Which of these lines is perpendicular to $4x - 5y = 7$?

A. $y = \frac{4}{5}x + \frac{4}{7}$

B. $y = -\frac{4}{5}x + 3$

C. $y = \frac{5}{4}x + \frac{6}{4}$

D. $y = -\frac{5}{4}x - 1$

E. None of them.

Example. Find an equation for a line PARALLEL to the line $4x - 6y + 1 = 0$ through the point $P(1, -5)$.

Example. Find an equation for a line PERPENDICULAR to the line $4x - 6y + 1 = 0$ through the point $P(1, -5)$.

Functions

After completing this section, students should be able to

- Decide whether a relationship between input and output values is a function or not, based on an equation, a graph, or a table of values.
- Find the corresponding output value for a given input value for a function given in equation, graphical, or tabular form.
- Find the corresponding input value(s) for a given output value for a function given in equation, graphical, or tabular form.
- Find the domains of functions given in equation form involving square roots and denominators.
- Find the domains and ranges of functions given in graphical or tabular form.
- Identify the graphs of the "toolkit functions" $y = x$, $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$, $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $y = 2^x$, $y = \log(x)$

Definition. A function is correspondence between input numbers (x-values) and output numbers (y-value) that sends each input number (x-value) to exactly one output number (y-value).

Sometimes, a function is described with an equation.

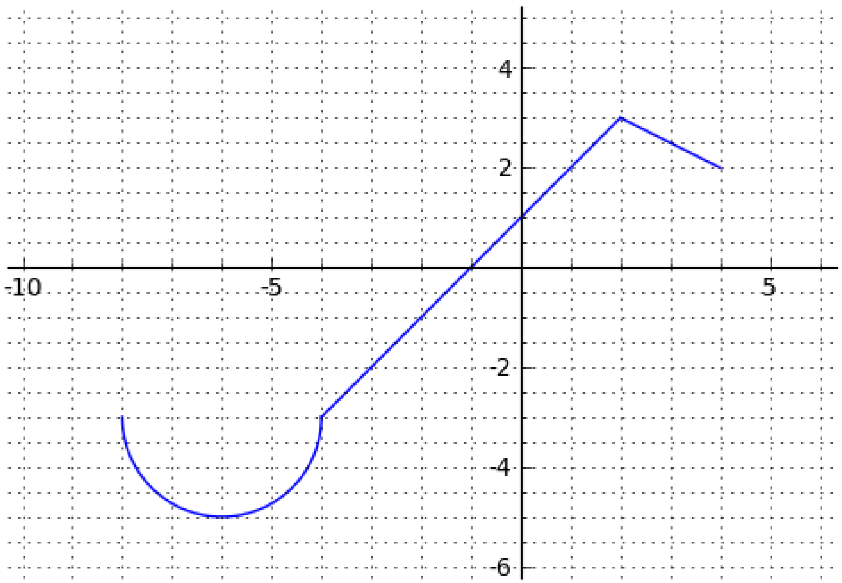
Example. $y = x^2 + 1$, which can also be written as $f(x) = x^2 + 1$

What is $f(2)$? $f(5)$?

What is $f(a + 3)$?

Sometimes, a function is described with a graph.

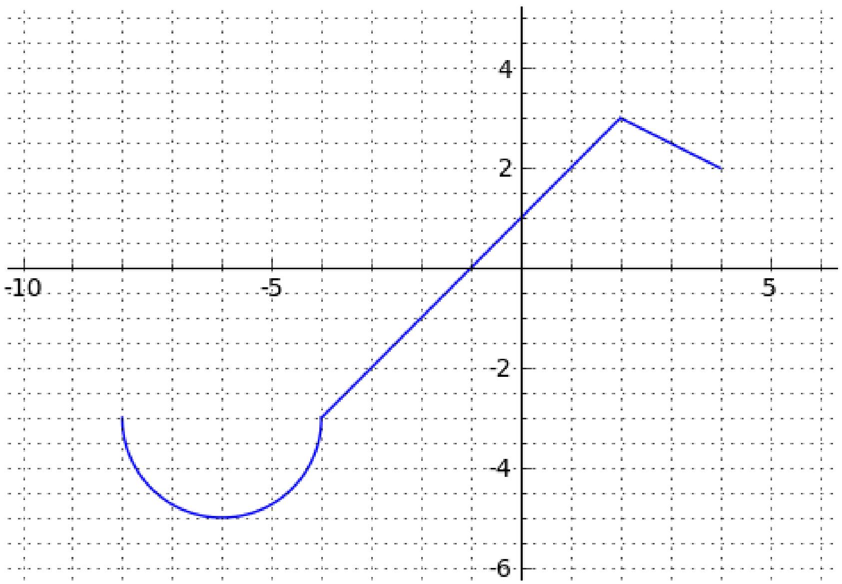
Example. The graph of $y = g(x)$ is shown below



What is $g(2)$? $g(5)$?

Definition. The *domain* of a function is all possible x-values. The *range* is the y-values.

Example. What is the domain and range of the function $g(x)$ graphed below?



Example. What are the domains of these functions?

A. $g(x) = \dfrac{x}{x^2 - 4x + 3}$

B. $f(x) = \sqrt{3 - 2x}$

C. $h(x) = \dfrac{\sqrt{3-2x}}{x^2 - 4x + 3}$

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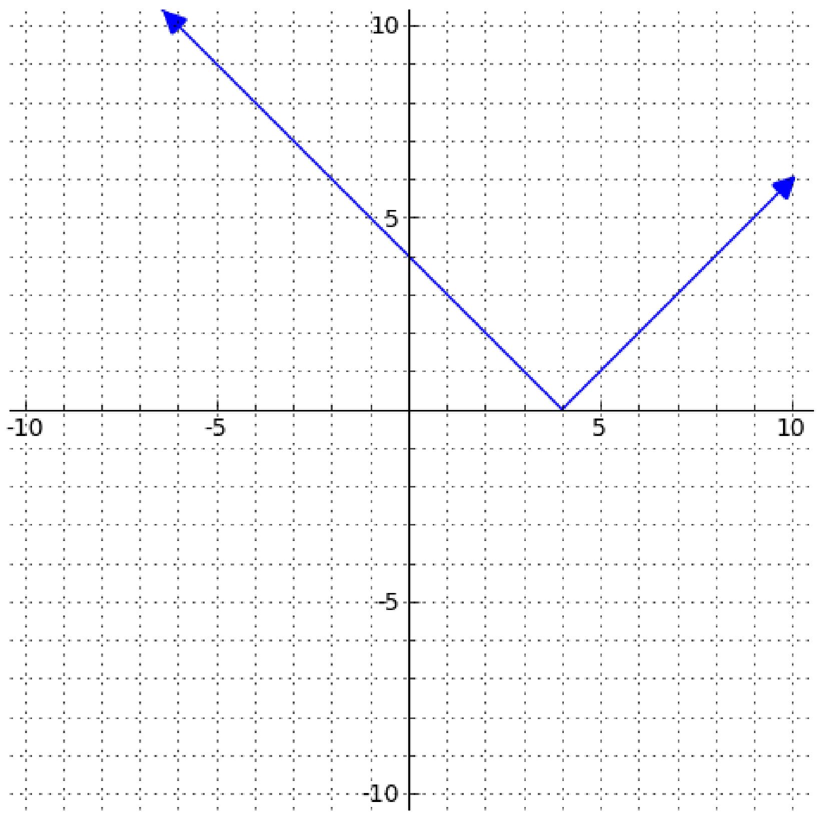
Example. The graph of $y = f(x)$ is drawn below.



1. Find $f(7)$.
2. Find all values a for which $f(a) = 5$.

Example. Suppose $f(x) = \dfrac{x+4}{x+1}$. Find all x such that $f(x) = 3$.

A. -1

B. $\frac{1}{2}$

C. $\frac{7}{4}$

D. 4

Example. Find the domain of $\sqrt{1-x^2}$. Write your answer in interval notation.

Example. Find the domain of $g(x) = \dfrac{\sqrt{3-x}}{x+1}$

A. $(-\infty, 3)$

B. $(-\infty, 3]$

C. $(-1, 3]$

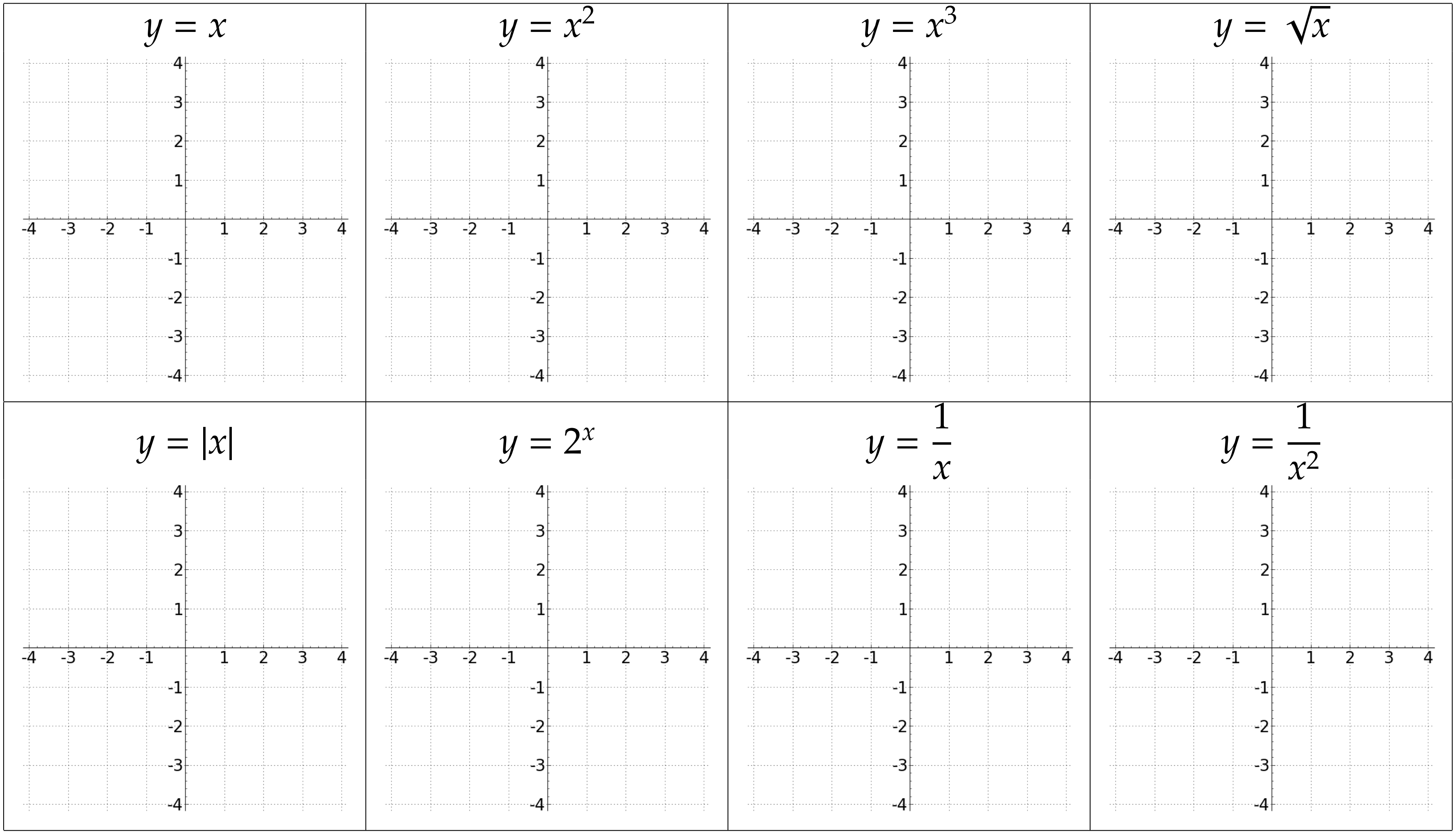
D. $(-\infty, -1) \cup (-1, 3)$

E. $(-\infty, -1) \cup (-1, 3]$

Toolkit Functions

After completing this section, students should be able to:

- Match the graphs of $y = x$, $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$, $y = 2^x$, $y = \log(x)$, $y = \frac{1}{x}$, and $y = \frac{1}{x^2}$ with their equations.
- Sketch approximate graphs of $y = x$, $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$, $y = 2^x$, $y = \log(x)$, $y = \frac{1}{x}$, and $y = \frac{1}{x^2}$ from memory, without having to plot individual points.



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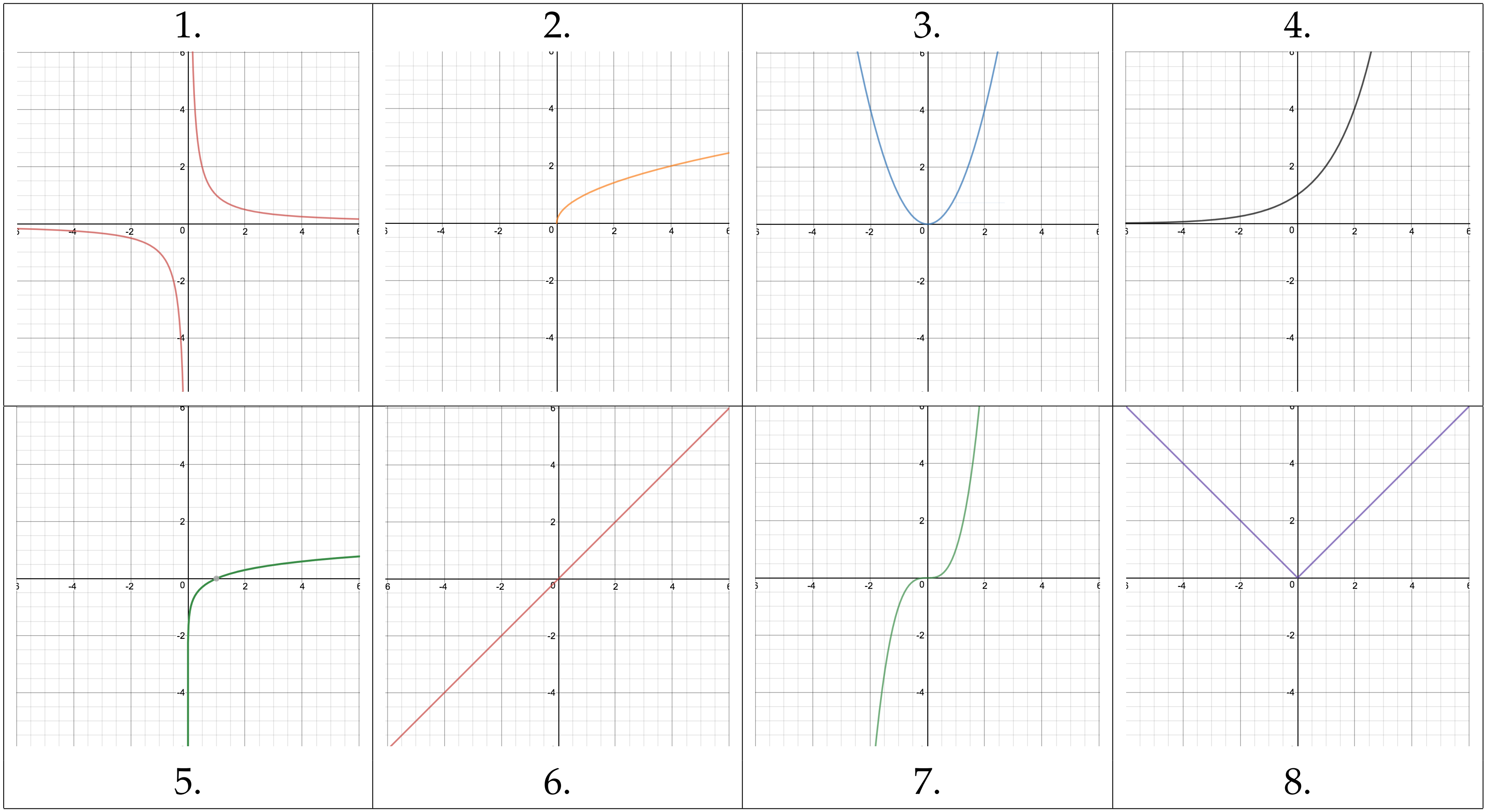
Example. Match the toolkit functions to the equations.



A. $y = x$ B. $y = x^2$ C. $y = x^3$ D. $y = \sqrt{x}$

E. $y = |x|$ F. $y = \dfrac{1}{x}$ G. $y = 2^x$ H. $y = \log(x)$

Transforming Functions

After completing this section, students should be able to

- Identify the motions corresponding to adding or multiplying numbers or introducing a negative sign on the inside or the outside of a function.
- Draw the transformed graph, given an original graph of $y = f(x)$ and an equation like $y = -3f(x+2)$, using a point by point analysis or a wholistic approach.
- Identify the equation for transformed graphs of toolkit functions like $y = |x|$ and $y = x^2$
- Identify a point on a transformed graph, given a point on the original graph and the equation of the transformed graph.

Review of Function Notation

Example. Rewrite the following, if $g(x) = \sqrt{x}$.

a) $g(x) - 2 =$

b) $g(x - 2) =$

c) $g(3x) =$

d) $3g(x) =$

e) $g(-x) =$

Example. Rewrite the following in terms of $g(x)$, if $g(x) = \sqrt{x}$.

f) $\sqrt{x} + 17 =$

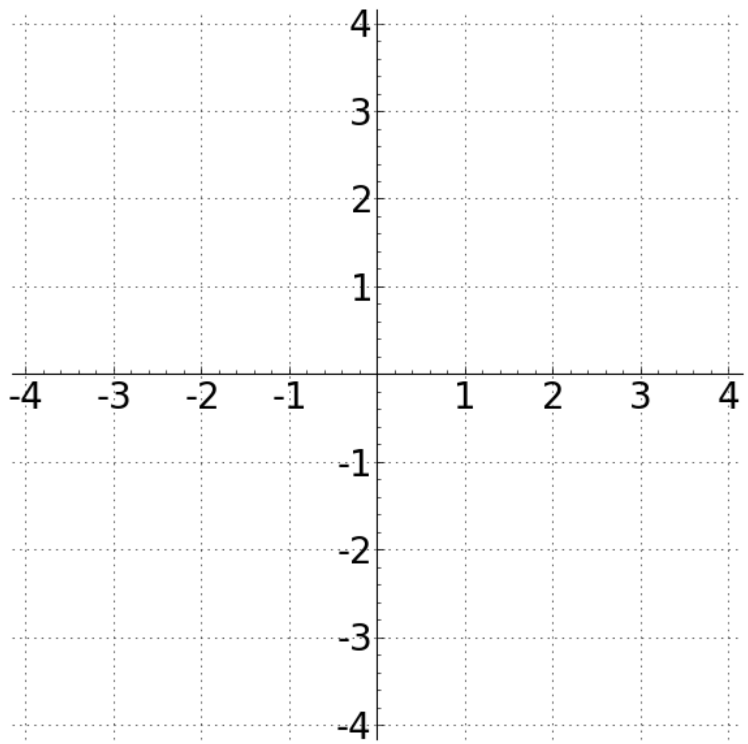
g) $\sqrt{x + 12} =$

h) $-36 \cdot \sqrt{x} =$

i) $\sqrt{\frac{1}{4}x} =$

Example. Graph

- $y = \sqrt{x}$
- $y = \sqrt{x} - 2$
- $y = \sqrt{x - 2}$



Rules for transformations:

- Numbers on the *outside* of the function affect the y-values and result in vertical motions. These motions are in the direction you expect.

- Numbers on the *inside* of the function affect the x-values and result in horizontal motions. **These motions go in the opposite direction from what you expect.**

- Adding results in a shift (translations)

- Multiplying results in a stretch or shrink

- A negative sign results in a reflection

Example. Consider $g(x) = \sqrt{x}$. How do the graphs of the following functions compare to the graph of $y = \sqrt{x}$?

a) $y = \sqrt{x} - 4$

b) $y = \sqrt{x + 12}$

c) $y = -3 \cdot \sqrt{x}$

d) $y = \sqrt{\frac{1}{4}x}$

END OF VIDEO

Rules of Function Transformations (see graph animations involving $y = \sqrt{x}$ and $y = \sin(x)$)

- A number added on the OUTSIDE of a function ...

- A number added on the INSIDE of a function ....

- A number multiplied on the OUTSIDE of a function ....

- A number mulitplied on the INSIDE of a function ....

- A negative sign on the OUTSIDE of a function ....

- A negative sign on the INSIDE of a function ....

Example. Consider $h(x) = x^2$. How do the graphs of the following functions compare to the graph of $y = x^2$?

a) $y = 3x^2$

b) $y = (x - \frac{7}{2})^2$

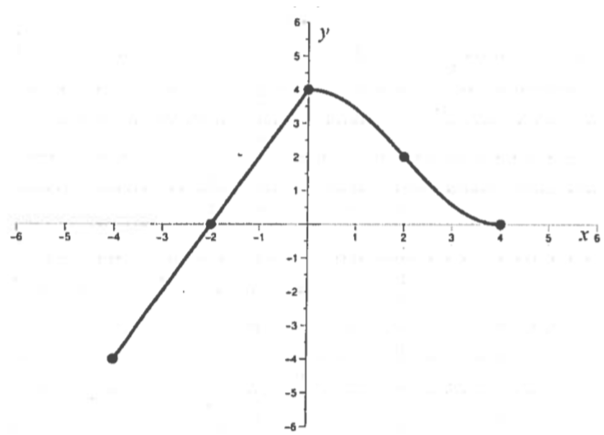
c) $y = x^2 + 5$

d) $y = (5x)^2$

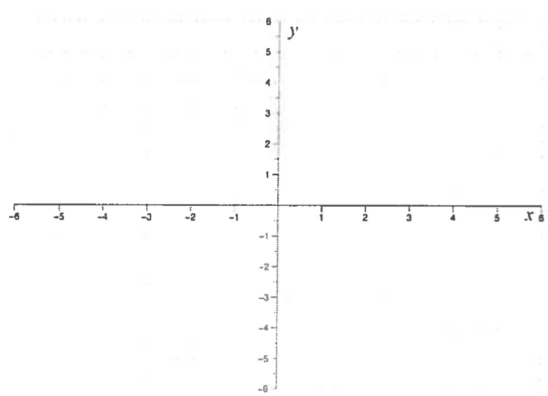
e) $y = -3(x - 2)^2 + 7$

Note. There are two approaches to graphing transformed functions:

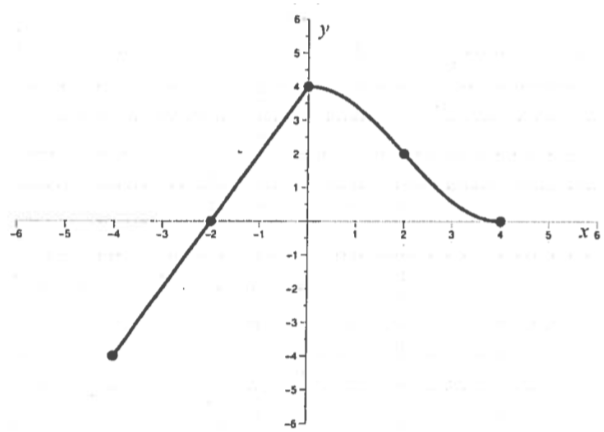
Example. The graph of a certain function $y = f(x)$ is shown below.



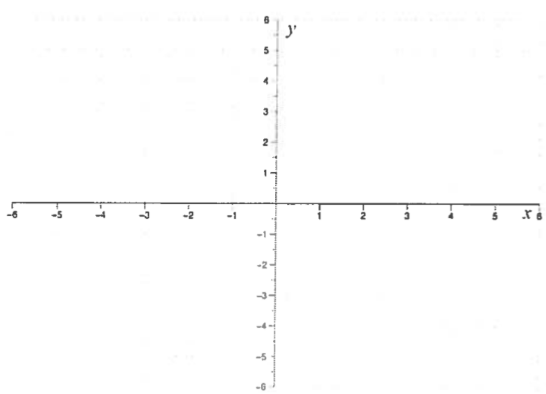
Use transformations to draw the graph of the function $y = -f(2x)$. Label at least 5 points on your final graph.



Example. The graph of a certain function $y = f(x)$ is shown below.



Use transformations to draw the graph of the function $y = f(x+1) + 2$. Label at least 5 points on your final graph.



Example. Suppose the graph of $y = f(x)$ contains the point $(3, -1)$. Identify a point that must be on the graph of $y = 2f(x-1)$.

A. $(2, -1)$

B. $(2, -1)$

C. $(4, -1)$

D. $(4, -2)$

Quadratic Functions

After completing this section, students should be able to:

- Find the x-intercepts and y-intercept or a quadratic function.
- Find the vertex of a quadratic function.
- Determine from the equation of a quadratic function if the graph will be pointing up or down.
- Graph a quadratic function from its equation.
- Convert a quadratic function from standard form to vertex form and vice versa.

Example. Which of these equations represent quadratic functions?

- $g(x) = -5x^2 + 10x + 3$
- $f(x) = x^2$
- $y = 3x - 2$
- $y = 2(x - 3)^2 + 4$

Example. Graph the following functions. For each graph label the vertex and the x-intercepts.

A. $f(x) = x^2$	B. $y = -3x^2$
C. $y = 2(x-3)^2 + 4$	D. $g(x) = 5x^2 + 10x + 3$

Summary

To graph a quadratic function $f(x) = ax^2 + bx + c$

- The graph has the shape ....

- The parabola opens up if ... and down if ...

- To find the x-intercepts ...

- To find the vertex ...

- To find additional points on the graph ....

Example. Convert this quadratic function to standard form: $f(x) = -4(x-3)^2 + 1$

Example. Convert this quadratic function to vertex form: $g(x) = 2x^2 + 8x + 6$

END OF VIDEOS

Example. What is the vertex for this quadratic function? $f(x) = 3(x+4)^2 - 1$

A. $(4, 1)$

B. $(4, -1)$

C. $(-4, 1)$

D. $(-4, -1)$

Example. Does the graph of this quadratic function open up or down? $y = -5x^2+40x-3$

A. Up

B. Down

Example. Find the vertex for the quadratic function $y = -5x^2 + 40x - 3$
Write your answer as an ordered pair.

Write the equation for this function in vertex form $y = a(x - h)^2 + k$

Example. Find the x-intercepts and the vertex for $y = 3x^2 + 7x - 5$.

Justification of the Vertex Formula Find the x-intercepts and vertex for $y = ax^2 + bx + c$.

Applications of Quadratic Functions

After completing this section, students should be able to:

- Write a quadratic equation to model a real world problem.
- Use a quadratic equation to answer questions involving maximums and minimums in a real world context.
- Use a quadratic equation to determine values of one variable based on given values of the other in a real world context.

Example. A projectile is launched straight up into the air so that its height in feet above the ground after t seconds is given by the function

$$h(t) = -16t^2 + 380t + 100$$

- How high does it go?

- When does it reach its maximum height?

- When does it hit the ground?

Example. A rancher plans to build a rectangular corral according to the plan shown in the diagram.

If the rancher has a total length of 800 feet of fencing, determine the dimensions that will allow for the maximum enclosed area.

Example. A rain gutter is formed by bending up the sides of a 30-inch wide and very long rectangular metal sheet. Where should we put the bends to maximize the amount of rain the gutter can carry?

Polynomials

After completing this section, students should be able to:

- Find the leading term, leading coefficient, and degree of a polynomial.
- Describe the relationship between the degree of the polynomial and the number of turning points.
- Predict the end behavior of the graph of a polynomial from the degree and the leading coefficient.
- Use the graph of a polynomial to determine what are all the possible options for degree and what the sign of the leading coefficient must be.
- Find the x- and y-intercepts of a rational function.
- Use the graph of a function to determine if a zero has multiplicity one or two.
- Write down the equation of polynomial from its graph.
- Match graphs of polynomials to equations.

Definition. The **degree** of the polynomial is

The **leading term** is

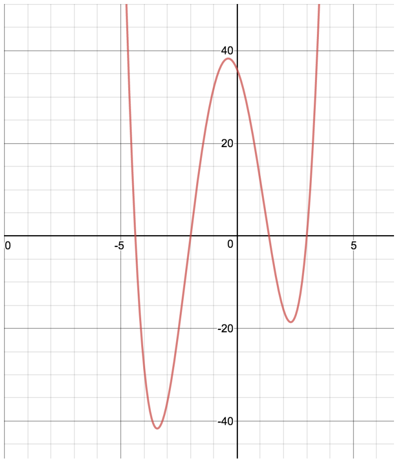
Definition. The **leading coefficient** is

Definition. The **constant term** is

Example. For $p(x) = 5x^3 - 3x^2 - 7x^4 + 2x + 18$, what is the

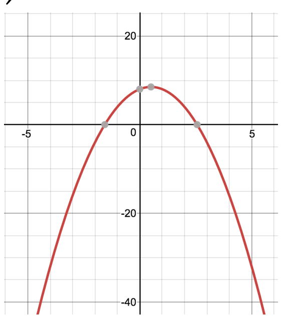
- degree?
- leading term?
- leading coefficient?
- constant term?

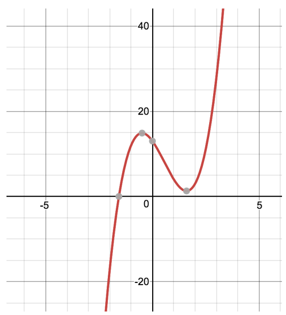
Definition. In the graph of $f(x) = x^4 + 2x^3 - 15x^2 - 12x + 36$ below, the marked points are called ...

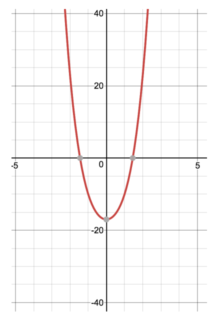


Compare the degrees of the polynomials to the number of turning points:

$f(x) = -2x^2 + 2x + 8$ $\quad f(x) = 3x^3 - 5x^2 - 7x + 13$ $\quad f(x) = x^4 + 6x^2 - 17$



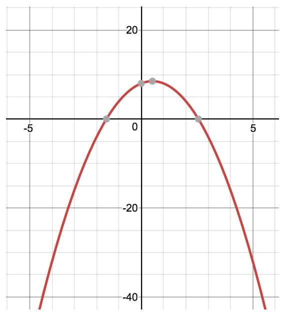




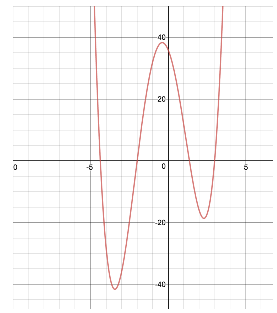
Definition. The **end behavior** of a function is how the "ends" of the function look as $x \to \infty$ and $x \to -\infty$.

Consider the end behavior for these polynomials:

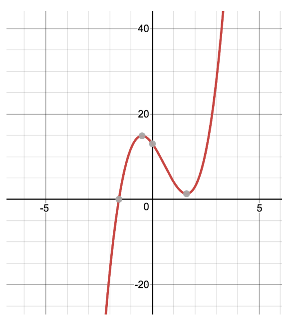
$$f(x) = -2x^2 + 2x + 8$$



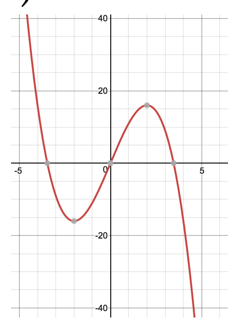
$$f(x) = x^4 + 2x^3 - 15x^2 - 12x + 36$$



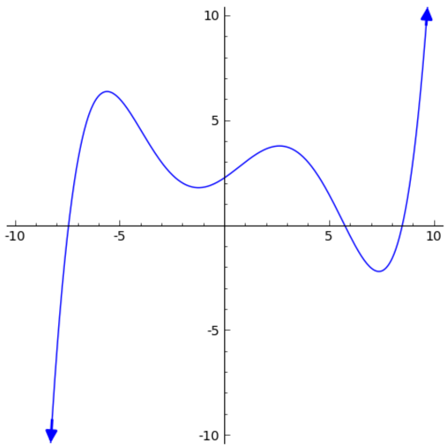
$$f(x) = 3x^3 - 5x^2 - 7x + 13$$



$$f(x) = -x^3 + 12x$$



Example. What can you tell about the equation for the polynomial graphed below?



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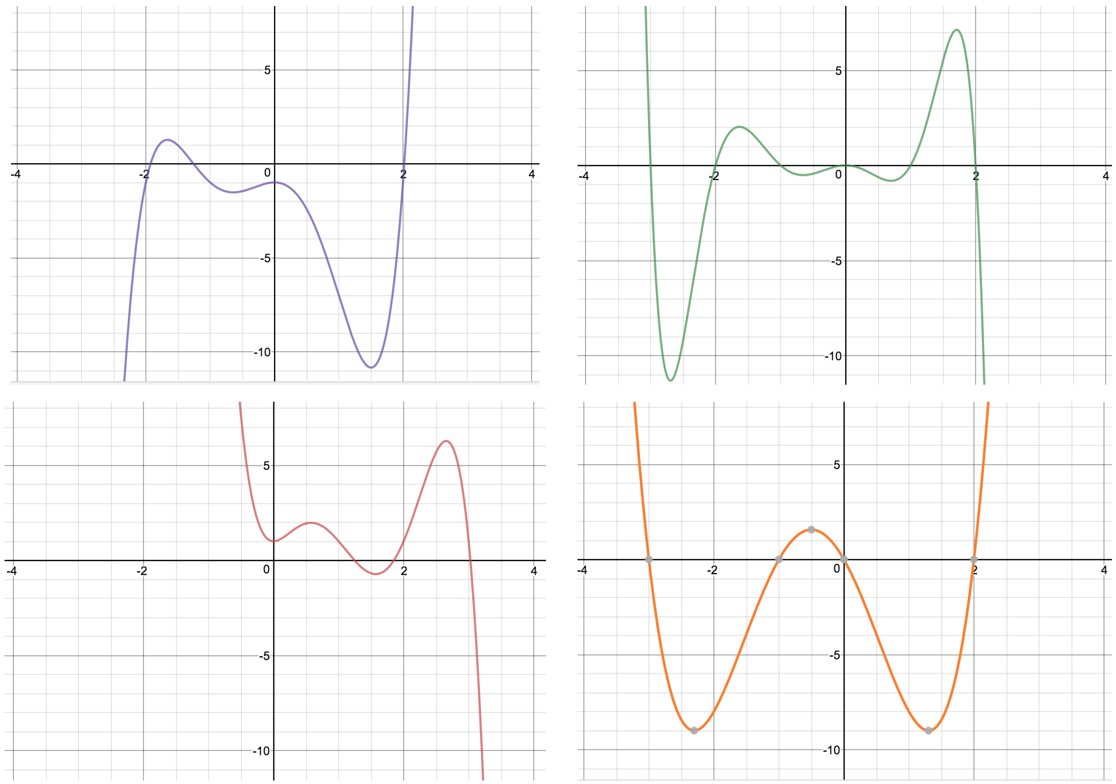
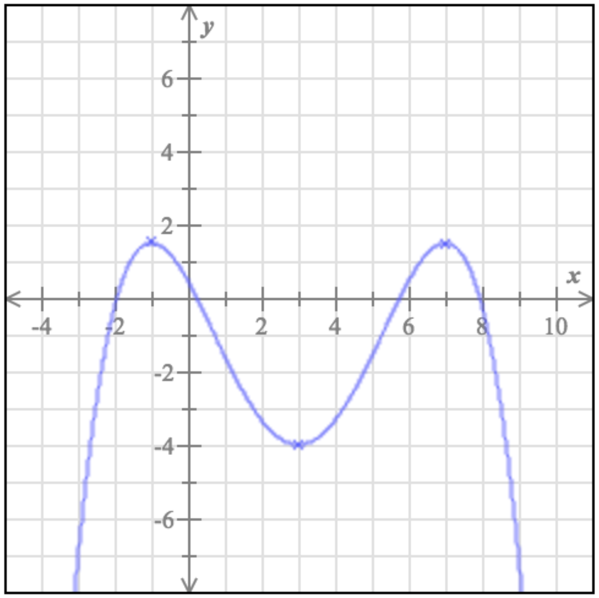
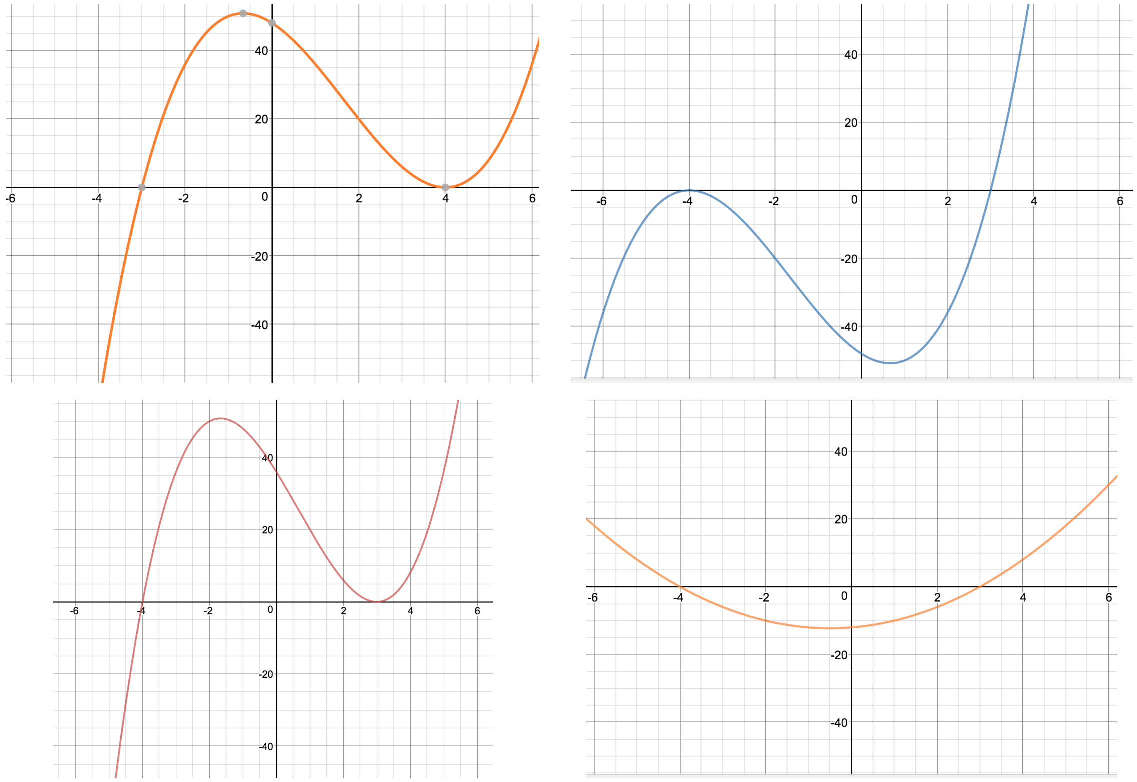
Example. Which figure shows the graph of $y = -2x^5 + 12x^4 - 22x^3 + 12x^2 + 1$?



Extra Example. The graph of a polynomial function $f(x)$ is shown below. What is the sign of the leading coefficient of f? What are the possible values for the degree of f?

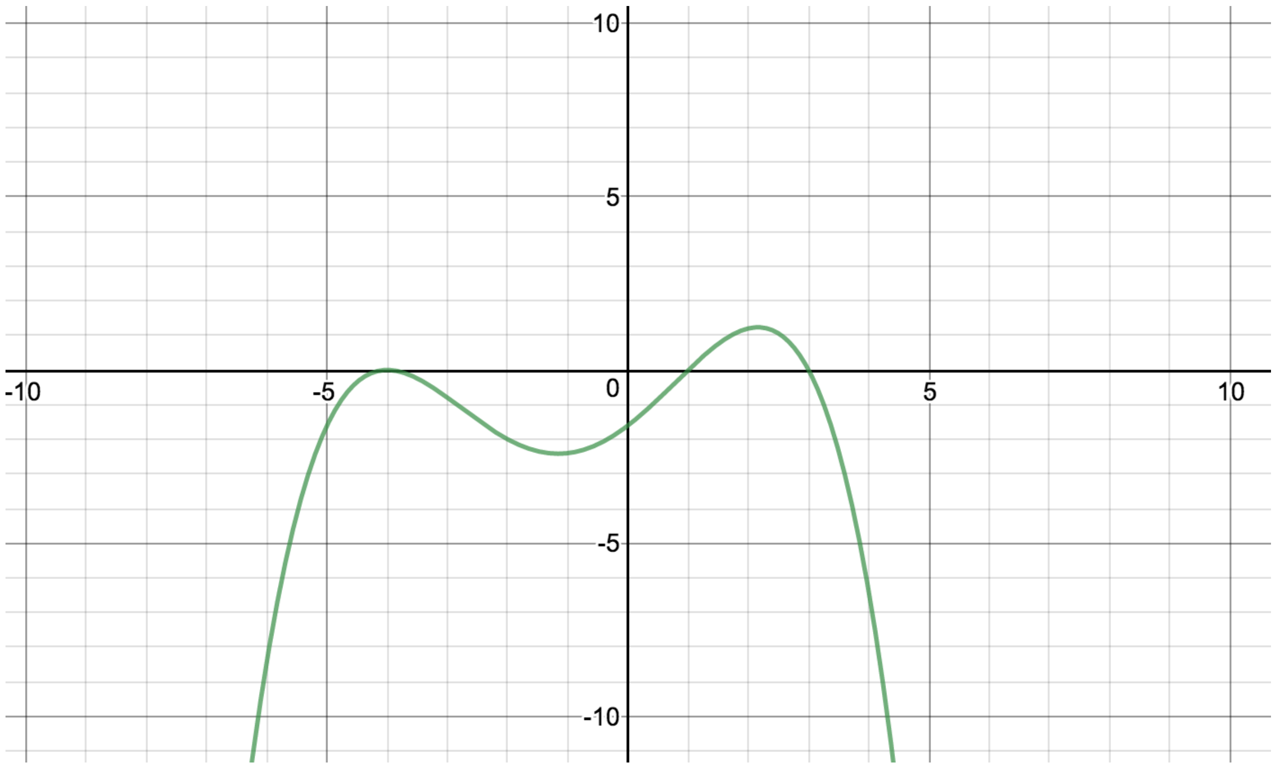


Example. Which figure shows the graph of $y = (x - 3)(x + 4)^2$?



Note. It is possible to use a number line to show that the graph of $y = (x - 3)(x + 4)^2$ "bounces" off the x-axis at $x = -4$ but not at $x = 3$.

Example. Find the equation for this graph:



Exponential Functions and Graphs

After completing this section, students should be able to

- Identify the domain, range, and horizontal asymptote of an exponential function from its graph or from its equation.
- For a function of the form, $y = a \cdot b^x$, explain how the values of a and b determine whether the graph is increasing or decreasing, how steeply it is increasing or decreasing, and what the value of its y-intercept is.
- Graph exponential functions and their transformations.
- Match the graphs of exponential functions to their equations.

Definition. An **exponential function** is a function that can be written in the form

$$f(x) = a \cdot b^x$$

where a and b are real numbers, $a \neq 0$, and $b > 0$.

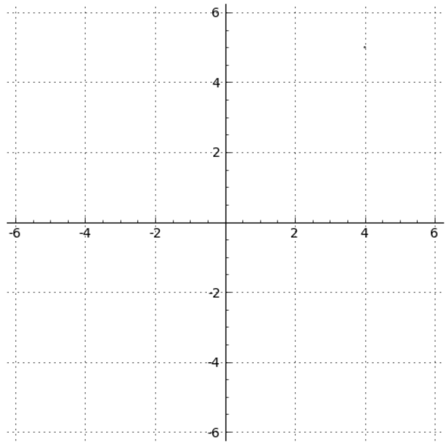
Note. We require that $a \neq 0$ because ...

Note. We require that $b > 0$ because ...

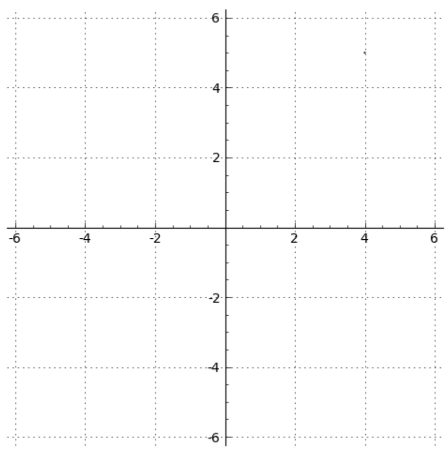
The number a is called the ...

and the number b is called the ...

Example. Graph the function $f(x) = 3 \cdot 2^x$



Example. Graph the function $g(x) = 3 \cdot \left(\frac{1}{2}\right)^x$.



Example. Graph the function $y = a \cdot b^x$ for different values of a and b simultaneously.

Fact. In the graph of $y = a \cdot b^x$:

- The parameter ________ gives the y-intercept.
- The parameter ________ tells how the graph is increasing or decreasing.
- If $b > 1$, the graph is ________________ .
- If $b < 1$, the graph is ________________.
- The closer b is to the number ________ , the flatter the graph.

So, for example, the graph of $y = 0.25^x$ is *(circle one)* flatter / more steep than the graph of $y = 0.4^x$.

Fact. The graph of an exponential function $y = a \cdot b^x$

- has a horizontal asymptote at the line ________________ .
- has domain ________________ .
- has range ________________ .

Note. The most famous exponential function in the world is $f(x) = e^x$. This function is sometimes written as $f(x) = \exp(x)$. The number e is Euler's number, and is approximately 2.71828182845904523...

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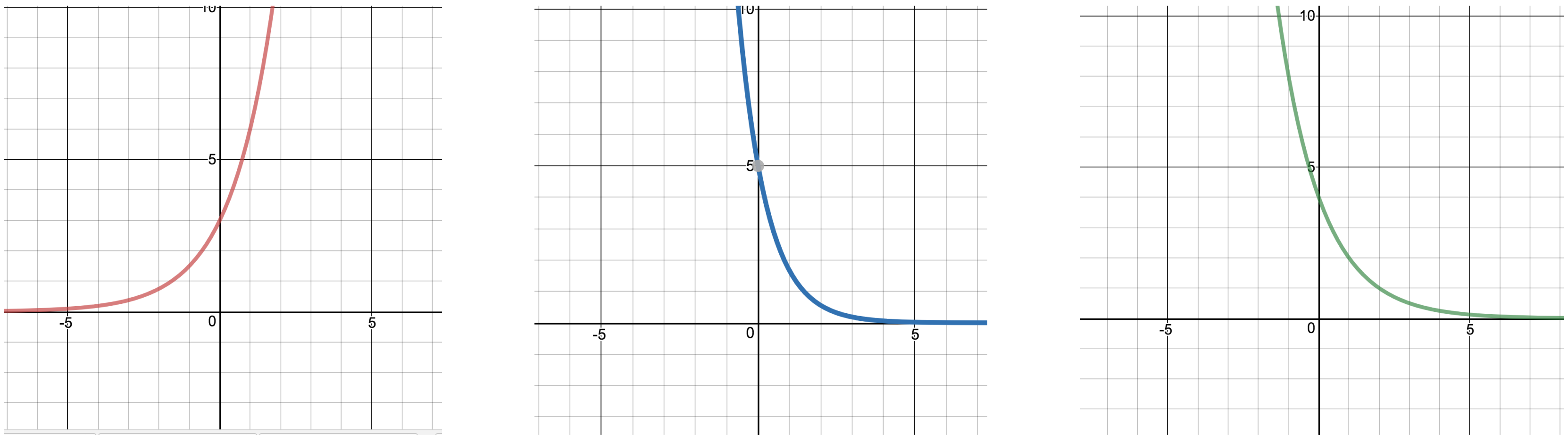
Question. What are some examples of exponential functions in the real world? Hint: what quantities have ″exponential growth″ or ″exponential decay″?

Example. Which graph represents which?



1. $y = 5 \cdot \left(\frac{1}{3}\right)^x$

2. $y = 3 \cdot 2^x$

3. $y = 4 \cdot 2^{-x}$

What is the domain, range, and horizontal asymptote for each of these three functions?

1. $y = 5 \cdot \left(\frac{1}{3}\right)^x$

2. $y = 3 \cdot 2^x$

3. $y = 4 \cdot 2^{-x}$

Summary: In the equation $y. = a \cdot b^x$, how do a and b affect the graph ? Consider:

- y-intercept
- x-intercept
- Increasing / decreasing
- Steepness
- Domain
- Range
- Horizontal asymptote(s)
- Vertical asymptote(s)

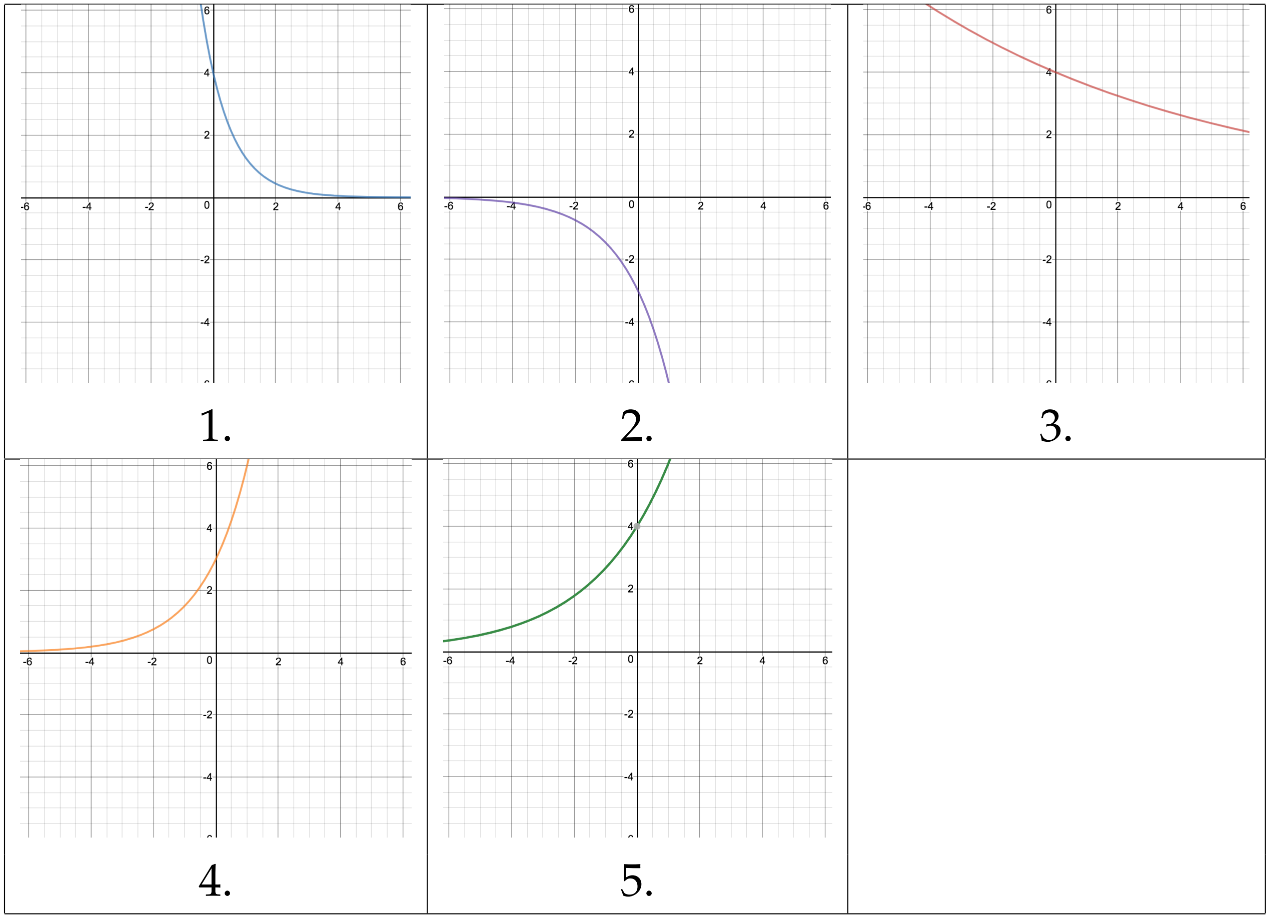
Example. For the following function, determine the domain, range, and horizontal asymptote:

$$y = 4 \cdot \left(\frac{1}{2}\right)^x - 6$$

Example. For the following function, determine the domain, range, and horizontal asymptote:

$$y = -\left(\frac{1}{4}\right)^{x+1} + 2$$

Example. Match the equations to the graphs.



A. $y = 4 \cdot 0.9^x$ B. $y = 4 \cdot 1.5^x$ C. $y = 4 \cdot 3^{-x}$ D. $y = 3 \cdot 2^x$
E. $y = -3 \cdot 2^x$

Applications of Exponential Functions

After completing this section, students should be able to:

- Write an exponential function to model exponential growth or decay.
- Given the equation of an exponential function, determine the initial value, the growth factor, and the percent by which the quantity is increasing or decreasing during each time unit.

Example. You are hired for a job and the starting salary is $40,000 with an annual raise of 3% per year. How much will your salary be after 1 year? 2 years? 5 years? t years?

Example. The United Nations estimated the world population in 2010 was 6.79 billion, growing at a rate of 1.1% per year. Assume that the growth rate stays the same. Write an equation for the population at t years after the year 2010.

Example. Seroquel is metabolized and eliminated from the body at a rate of 11% per hour. If 400 mg are given, how much remains in the body after 24 hours?

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Example. During the Ebola epidemic in 2014, the total number of Ebola cases in Guinea, Sierra Leone, and Liberia was increasing by 25% every week. By June 1, there were 528 reported cases. Which of the following functions represents the total number of Ebola cases t weeks after June 1?

A. $f(t) = 528 + 25t$

B. $f(t) = 528 + 0.25t$

C. $f(t) = 528 \cdot 25^t$

D. $f(t) = 528 \cdot (0.25)^t$

E. $f(t) = 528 \cdot (1.25)^t$

Based on this model, how many cases do you expect 32 weeks after June 1?

Example. The USDA reports that on Jan 1, 2016, there were 2.59 million honey bee colonies, which was an 8% drop from the number one year previously. If the number of honey bee colonies continues to drop by 8% each year, which function represents the number of honey bee colonies t years after Jan 1, 2016?

A. $f(t) = 2.59 \cdot (-0.08)^t$

B. $f(t) = 2.59 \cdot (0.08)^t$

C. $f(t) = 2.59 \cdot (0.92)^t$

D. $f(t) = 2.59 \cdot (1.08)^t$

Example. Suppose the price of a 1 terabyte external hard drive is given by the function $P(t) = 65 \cdot 0.84^t$ where t is time in years since 2016 and $P(t)$ is price in dollars.

1. Is the price of the hard drive increasing or decreasing?

2. At what rate is it increasing or decreasing? Give your answer as a percent.

3. What was the price in 2016?

Extra Example. The number of opioid overdose deaths per 100,00 people in the US is approximated by the equation $D(t) = 9.0 \cdot 1.14^t$, where t is the number of years since 2014 and $D(t)$ is the number of deaths per year.

1. How many opioid overdose deaths were there in 2014?

2. Is the number of deaths increasing or decreasing?

3. At what rate is the number of deaths changing?

4. How many opioid overdose deaths do you expect per 100,000 people in 2018?

Compound Interest

After completing this section, students should be able to:

- Explain where the formula for compound interest comes from.
- Solve for the final amount of money from the initial amount of money, or vice versa, given information about the interest rate, the compounding period, and the number of years.
- Solve for the final amount of money from the initial amount of money, or vice versa, for continuously compounded interest, given information about the interest rate and the number of years.
- Compare the money accumulated when interest is compounded at different time periods and continuously.

Example. Suppose you invest \$ 200 in a bank account that earns 3% interest every year. If you make no deposits or withdrawals, how much money will you have after 10 years?

Example. You deposit \$300 in an account that earns 4.5% annual interest compounded *semi-annually*. How much money will you have 7 years?

Example. You take out a loan for \$1,200 at an annual interest rate of 6%, compounded *monthly*. If you pay back the loan with interest as a lump sum, how much will you owe after 3 years?

Example. You invest \$4000 in an account that gives 2.5% interest compounded *continuously*. How much money will you have after 5 years?

Summary:

- Let r represent ...
- Let t represent ...
- Let A represent the initial amount of money ...

Annual interest:

Compound interest, compounded n times per year:

Compound interest, compounded continuously.

END OF VIDEO

Example. Match the equations with their descriptions:

1. $y = Pe^{rt}$
2. $y = P(1 + r)^t$
3. $y = P\left(1 + \frac{r}{n}\right)^{nt}$
4. $y = a \cdot b^t$

A. Money earns 5% interest once a year.

B. Money earns 2% interest compounded monthly.

C. Money earns interest compounded continuously, at an APR of 4%.

D. A population grows at a rate of 3% annually.

E. A population grows at a continuous rate of 7% per year.

Question. If you invest \$1000, how much money do you have after a year,

(a) at a 5% APR compounding annually?

(b) at a 5% APR compounded monthly?

(c) at a 5% APR compounded daily?

(d) at a 5% compounded continuously?

Question. What is the APY (annual percentage yield) in each of these cases?

Extra Example. Your grandparents are loaning you \$5000 to buy a car. Instead of making monthly payments, they ask that you pay back all the money in 10 years, with interest, all as a lump sum. They give you three options:

Option 1: Annual interest rate of 9.0% compounded once a year.

Option 2: Annual interest rate of 8.95%, compounded monthly.

Option 3: Annual interest rate of 8.95%, compounded continuously.

Which is best for you? Which is worst?

Continuous Growth

After completing this section, students should be able to:

- Write an equation to describe other quantities, like population, that grow or shrink continuously.
- Compare and contrast the formulas $y = a \cdot b^t$ and $a \cdot (1 + r)^t$ for exponential growth and the formula $y = a \cdot e^{rt}$ for continuous exponential growth and explain why the two formulas yield different results when the same value of a and r is used in both.

Example. Write equations to describe each of the following populations:

a) A population grows at a **continuous rate** of 6% per year.

b) A population shrinks at a **continuous rate** of 10.5% per year.

Example. Which population is growing fastest?

a) A population that grows at a **continuous rate** of 19% per year.

b) A population that grows at a rate of 19% per year.

For the population that grows at a continuous rate of 19% per year, by what percent does it actually increase at the end of each year?

Compare and contrast the formulas for:

Exponential Growth | Continuous Exponential Growth

Example. Rewrite the equation $y = 40e^{0.17t}$ in the form $y = a \cdot b^t$ and in the form $y = P(1 + r)^t$.

Question. Could we rewrite the equation $y = 88 \cdot 1.07^t$ in the form $y = Pe^{rt}$?

Logarithms

After completing this section, students should be able to:

- Rewrite a log equation as an exponential equation and vice versa.
- Compute simple logarithms by hand by expressing the logarithm expression as a question about exponents.
- Explain how to simplify expressions like $\log_4 4^z$ and $5^{\log_5 t}$ and why this makes sense.
- Explain why logs are useful.

Definition. $\log_a b = c$ means $a^c = b$.

You can think of logarithms as exponents: $log_a b$ is the exponent (or "power") that you have to raise a to, in order to get b. The number a is called the *base* of the logarithm. The base is required to be a positive number.

Example.

$$\log_2 8 = 3 \text{ because } 2^3 = 8$$

$$\log_2 y = \square \text{ means } 2^\square = y$$

Example. Evaluate the following expressions by hand by rewriting them using exponents instead of logs:

a) $\log_2 16 =$________

b) $\log_2 2 =$________

c) $\log_2 \frac{1}{2} =$________

d) $\log_2 \frac{1}{8} =$________

e) $\log_2 1 =$________

Example. Evaluate the following expressions by hand by rewriting them using exponents instead of logs:

a) $\log_{10} 1,000,000 =$ ________

b) $\log_{10} 0.001 =$ ________

c) $\log_{10} 0 =$ ________

d) $\log_{10} -100 =$ ________

Note. It is possible to take the log of numbers that are ________ but not of numbers that are ________ . In other words, the domain of the function $f(x) = \log_a(x)$ is: ________________ .

Note. $\ln x$ means $\log_e x$, and is called the **natural log**.

$\log x$, with no base, means $\log_{10} x$ and is called the **common log**.

You can find $\ln x$ and $\log x$ for various values of x using the buttons on your calculator.

Example. Rewrite using exponents.

a) $\log_3 \frac{1}{9} = -2$

b) $\log 13 = 1.11394$

c) $\ln \frac{1}{e} = -1$

Example. Rewrite the following using logs. Do not solve for any variables.

a) $3^u = 9.78$

b) $e^{3x+7} = 4 - y$

END OF VIDEO

What are logs?

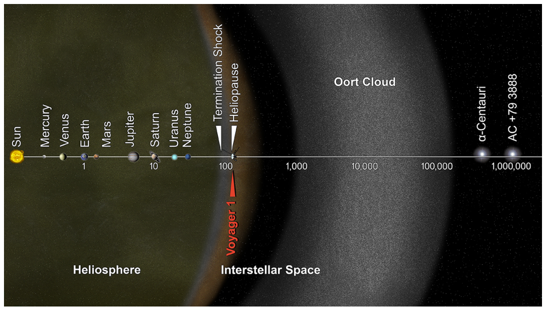
A way of writing exponents:

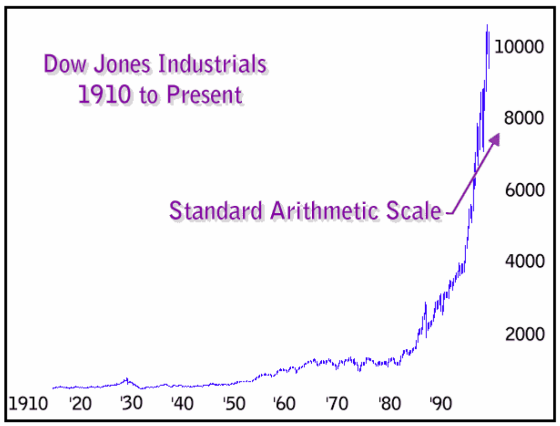
$\log_2 8$ means ...

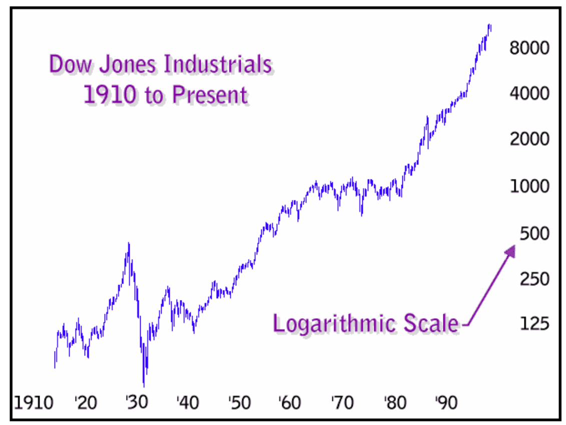
$\log_2 8 = 3$ because ...

What are logs good for?

- Working with quantities of wildly different sizes.







- Solving equations with variables in the exponents.

$$y = Pe^{rt} \qquad\qquad y = 100e^{0.05t}$$

Example. What is $\log_5 125$? What is $\log_5 \frac{1}{5}$?

Example. Rewrite the log expression as an exponential expression: $\log_3(2y) = 4$.

Example. Rewrite the exponential expression in terms of logs: $e^{4u} = 5$

Extra Example. Evaluate $\log_4 4^3$

Extra Example. What is $\log_6 6^x$?

Extra Example. Evaluate $2^{\log_2 32}$

Extra Example. Evaluate $3^{\log_3 \frac{1}{9}}$

Extra Example. Evaluate $5^{\log_5 57}$

Extra Example. Evaluate $2^{\log_2(a+6)}$.

Extra Example. Rewrite as an exponential equation and solve for x:

$$\log_{x+3} 0.01 = 2$$

Log Functions and Graphs

After completing this section, students should be able to:

- Recognize the graphs of log functions.
- Graph a basic log function by hand.
- Find the asymptote and intercepts of log functions like $y = \log_2(x)$ and transformed log functions like $y = \log(x + 3)$.
- Find the domains of functions involving logs.

Example. Graph $y = \log_2(x)$ by plotting points.

Example. Graph $y = \ln(x) + 5$. Find the domain, range, and asymptotes.

Example. Graph $y = \log(x + 2)$. Find the domain, range, and asymptotes.

Find the domain of $f(x) = \ln(2 - 3x)$.

END OF VIDEO

Example. Graph

$y = \log_2(x)$ $y = \log_2(x)$ $y = \ln(x)$

What is the

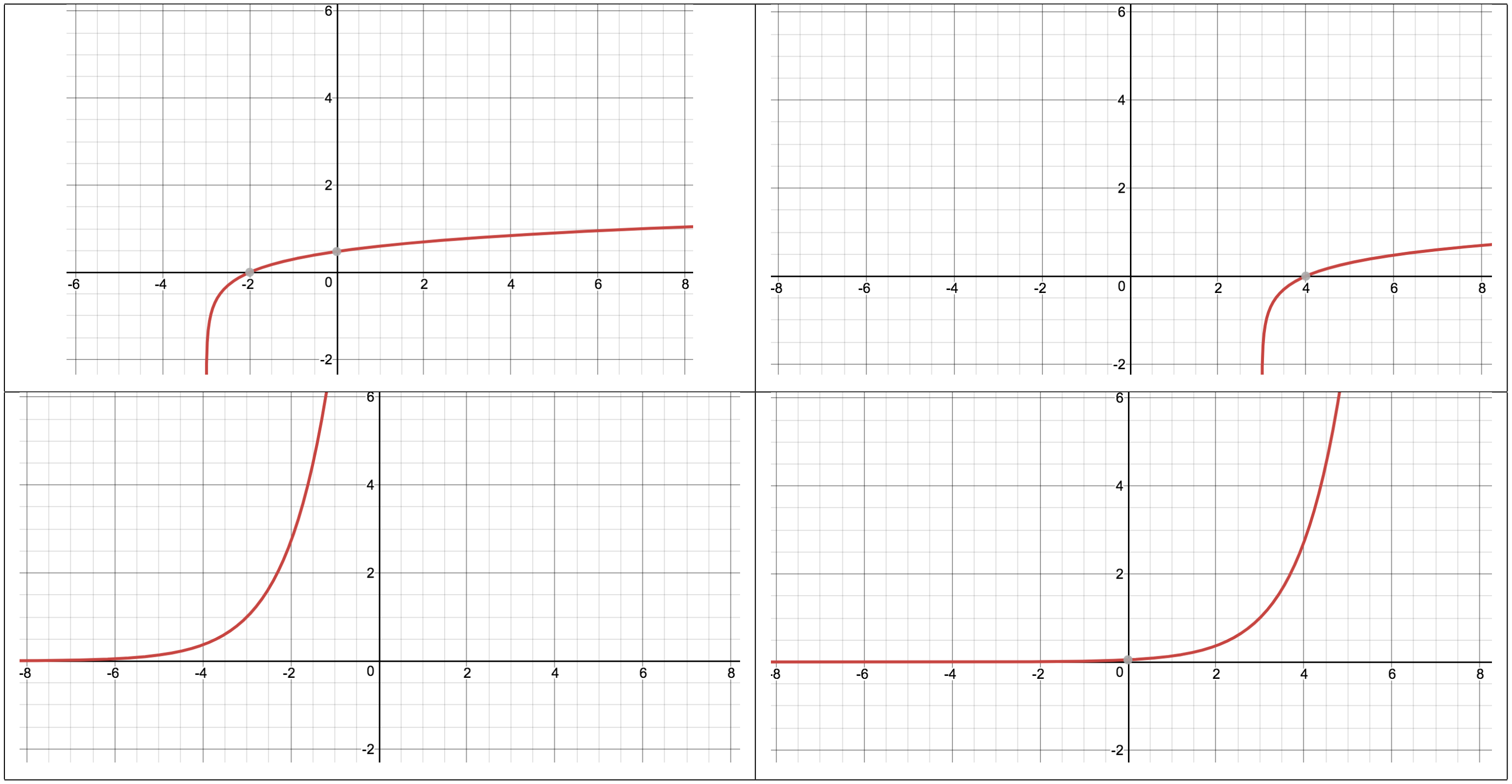
- domain

- range

- asymptote(s)

- intercept(s)

Which one of these graphs represents $y = \log(x + 3)$?



Where does the graph of $y = \log(x)$ have an asymptotes? What about $y = \log(x + 3)$?

What is the domain of $y = \log(x)$? What about $y = \log(x + 3)$?

What is the domain of $f(x) = \log(x^2 - 1)$?

Combining Logs and Exponents

After completing this section, students should be able to:

- Recognize that $\log_a a^b = b$ and explain why this is true.
- Recognize that $a^{\log_a b} = b$ and explain why this is true.

Example. Evaluate:

a) $\log_{10} 10^3$

b) $\log_e e^{4.2}$

c) $10^{\log_{10} 1000}$

d) $e^{\log_e 9.6}$

Log Rule: For any base a, $\log_a a^x =$

Log Rule: For any base a, $a^{\log_a x} =$

Example. Find $3^{\log_3 1.4}$

Example. Find $\ln(e^x)$

Example. Find $10^{\log(3z)}$

Example. True or False: $\ln 10^x = x$.

END OF VIDEO

Example. Simplify the following expressions:

A. $\log_4 4^3$

B. $\log_2 2^{-5}$

C. $4^{\log_4 \frac{1}{64}}$

D. $7^{\log_7 18}$

E. $\ln e$

F. $e^{\ln 5}$

Extra Example. Evaluate $\log_6 6^{2x+7}$

Extra Example. Evaluate $2^{\log_2(a+6)}$.

Log Rules

After completing this section, students should be able to:

- Write down the product, quotient, and power rules for logs as equations, and describe them in words.
- Explain how the product, quotient, and power rules for logs are related to the corresponding rules for exponents.
- Use the log rules to expand the log of a complicated expression into a sum or difference of multiplies of logs.
- Use the log rules to condense an expression involving a sum or difference of multiples of logs into a single log.

Note. Recall some of the exponent rules.

1.

2. Product rule:

3. Quotient rule:

4. Power rule:

Note. The exponent rules hold for any base, not just base 2.

The corresponding logarithm rules are:

1.

2. Product rule:

3. Quotient rule:

4. Power rule:

Note. The logarithm rules hold for any base, not just base 2.

Exponent Rule	Log Rule	Name of Log Rule
$a^0 = 1$		–
$a^m \cdot a^n = a^{m+n}$		
$\frac{a^m}{a^n} = a^{m-n}$		
$(a^m)^n = a^{mn}$		

Example. Rewrite the following as a sum or difference of logs:

a) $\log\left(\frac{x}{yz}\right)$

b) $\log(5 \cdot 2^t)$

Example. Rewrite as a single log:

a) $\log_5 a - \log_5 b + \log_5 c$

b) $\ln(x + 1) + \ln(x - 1) - 2\ln(x^2 - 1)$

END OF VIDEO

Review. $\log_a a^b =$

Review. $a^{\log_a b} =$

Note. Which order of operations is the correct interpretation for $\log_2 3^7$?

(a) $\log_2(3^7)$

(b) $(\log_2 3)^7$

Name of Rule	Exponent Rule	Log Rule	Description of Log Rule in Words
Zero Power	$3^0 =$		
Product Rule	$3^a 3^b =$		
Quotient Rule	$\frac{3^a}{3^b} =$		
Power Rule	$(3^a)^b =$		

Example. Rewrite these expressions using log rules.

(a) $\log\left((x \cdot y)^4\right) =$

(b) $\log\left(x \cdot y^4\right) =$

Example. Use the properties of logs to expand the following expression:

$$\log\left(\frac{\sqrt[3]{y^4}}{xz^4}\right)$$

Example. Use properties of logs to expand the following expression:

$$\log\left(\frac{(x+4)^5}{\sqrt{x^3}}\right)$$

A. $\frac{(\log(x+4))^5}{\sqrt{(\log(x))^3}}$

B. $\frac{5\log(x+4)}{\frac{3}{2}\log(x)}$

C. $5\log(x+4) - \frac{3}{2}\log(x)$

D. $5\log(x) + 5\log(4) - \frac{3}{2}\log(x)$

Example. Write the expression as a single log:

$$3\ln(x) - \ln(x-7) + \frac{\ln(x+1)}{2}$$

Example. Write the expression as a single log:

$$\log_3(x+2) - \log_3(y) - \frac{1}{3}\log_3(z-1)$$

A. $\log_3\left(\frac{(x+2)}{\frac{1}{3}y(z-1)}\right)$

B. $\log_3\left(\frac{(x+2)}{y(z-1)^{1/3}}\right)$

C. $\log_3\left(\frac{(x+2)(z-1)^{1/3}}{y}\right)$

D. $\log_3\left(x+2-y-(z-1)^{1/3}\right)$

Example. Remembering the log rules, decide which of the following statements are true. (Select all correct answers, and assume all arguments of ln are positive numbers.)

A. $\ln(x + 1) + \ln(x - 1) = \ln(x^2 - 1)$

B. $\ln(x) - \ln(y) = \frac{\ln(x)}{\ln(y)}$

C. $\ln(9x + 17) = \ln(9x) + \ln(17)$

D. $\frac{\ln(x^2)}{\ln(x)} = 2$ for $x \neq 1$

E. $\ln(5x^3) = 3\ln(5x)$

Extra Example. Remembering the log rules, decide which of the following statements are true. (Select all correct answers, and assume all arguments of ln are positive numbers.)

A. $\log(\frac{1}{a}) = -\log(a)$

B. $10^{2\log x} = x^2$

C. $\log(w - v) = \frac{\log(w)}{\log(v)}$

Solving Exponential Equations

After completing this section students should be able to:

1. Use logs to solve equations with variables in the exponent.

Example. Solve: $5 \cdot 2^{x+1} = 17$

Example. Solve: $2^{2x-3} = 5^{x-2}$

Example. Solve: $5 \cdot e^{-0.05t} = 3 \cdot e^{0.2t}$

END OF VIDEO

Example. In 2015, the population of Guatemala was 16.3 million and was growing at a rate of 2.1% per year.

1. Write an equation to model the population of Guatemala, assuming this rate of growth continues. Let t represent the number of years since 2015.

2. If this rate continues, when will the population of Guatemala reach 100 million?

Example. In 2015, the population of Nigeria was 182 million and was growing at a **continuous** rate of 2.7% per year.

1. Write an equation to model the population of Nigeria, assuming this rate of growth continues. Let t represent the number of years since 2015.

2. If this rate continues, when will the population of Nigeria reach 1 billion?

Example. Solve for x: $11^{-x+2} = 14^{-10x}$

Example. Solve for x: $4^{2-3x} = 5^x$

Solving Log Equations

After completing this sections, students should be able to:

- Solve equations with log expressions in them.

Example. Solve: $2\ln(2x+5)-3=1$

Example. Solve: $\log(x + 3) + \log(x) = 1$

Example. Solve: $\log_3 5 = x$

Example. Solve: $4 \cdot x^6 - 1 = 18$

END OF VIDEO

Example. Solve for x:

$$3\log_2(x+3) - 1 = 6$$

Example. Solve for x: $\log(x+2) + \log(x-1) = 1$

Doubling Time and Half Life

After completing this section, students should be able to:

- Define doubling time and half life.
- Calculate doubling time of half life from an equation for exponential growth or decay or from a growth or decay rate.
- Use double time or half life to write an equation to model exponential growth or decay in a real world applications.

Example. Suppose you invest \$1600 in a bank account that earns 6.5% annual interest, compounded monthly. How many years will it take until the account has \$2000 in it, assuming you make no further deposits or withdrawals?

Example. A population of bacteria contains 1.5 million bacteria and is growing by 12% per day. What is its *doubling time*?

Example. Suppose a bacteria population doubles every 15 minutes. Write an equation for its growth using the exponential equation $y = a \cdot b^t$, where t represents time in minutes.

Example. The half life of radioactive Carbon-14 is 5750 years. A sample of bone that originally contained 200 grams of C-14 now contains only 40 grams. How old is the sample?

END OF VIDEO

Example. Suppose today, your grandparents put \$10,000 in a bank account in your name, earning 4.5% annual interest, compounded monthly. How long will it take for that money to double (assuming no money is added or withdrawn)?

What if the original deposit is \$20,000 ?

Example. The doubling time of a bacteria population is 18 hours. How long will it take for the bacteria culture to grow from 5 grams to 12 grams?

Example. Tylenol has a half life of approximately 2.5 hours. If you take Tylenol once and then repeat the dose again after 4 hours, how much of the original dose is still in your system when you take the second dose?

Summary:

- What are the equations we use to model exponential growth and decay (including interest rate problems)?
- In what situations do we use each type?
- In what situations can we freely choose between two or more equations?

Example. A car that was worth \$15,000 in 2015 is now worth \$11,000.

A. Assuming an exponential decay model, what will the car be worth in 2025?

B. Assuming a linear model, what will the car be worth in 2025?

Example. Write an equations to model the following situations:

1. In the year 2000, the population of raccoons on Island A was 70 thousand and increasing at a rate of 5% per year. Write an equation to model the raccoon population over time, where t represents years since 2000.

2. In the year 2000, the population of raccoons on Island B was 70 thousand and increasing at a rate of 5 thousand raccoons per year. Write an equation to model the raccoon population over time, where t represents years since 2000.

Extra Example. In 1991 hikers found the preserved body of a man partially frozen in a glacier in the Austrian Alps. It was found that the body of Otzi (as the iceman came to be called) contained 53% as much Carbon-14 as the body of a living person. What is the approximate date of his death?

Solving Systems of Equations

After completing this section, students should be able to:

- Determine if an (x, y) pair is the solution to a system of equations in two variables.
- Use substitution to solve a system of linear equations in two variables.
- Use elimination to solve a system of linear equations in two variables.
- Determine algebraically if a system of linear equations has one solution, no solutions, or infinitely many solutions.
- Explain how graphing the two linear equations in a system of equations can show whether the system will have one solution, no solutions, or infinitely many solutions.
- Use substitution or elimination to solve a system of non-linear equations in two variables.

Example. Solve the system of equations:

$$3x - 2y = 4$$
$$5x + 6y = 2$$

Example. Solve the system of equations:

$$8y = 1 + 4x$$
$$3x - 6y = 2$$

Example. Solve the system of equations:

$$x + 5y = 6$$
$$3x + 15y = 18$$

END OF VIDEO

Example. Solve for x and y:

$$\frac{1}{5}x = \frac{1}{4}y - 6$$
$$-\frac{3}{4}x + \frac{1}{2}y = -2$$

Example. Which system of equations has NO solutions?

A. $x + y = 1, 2x + y = 3$

B. $x + y = 1, 2x + 2y = 4$

C. $x + y = 1, 2x + 2y = 2$

Example. Solve for x and y:

$$x^2 = y - 10$$
$$3x + y = 14$$

Example. Solve for x and y:

$$x^2 + y^2 = 25$$
$$x^2 - 2y = 1$$

Applications of Systems of Equations

After completing this section, students should be able to:

- Set up and solve systems of equations involving distance, rate, and time.
- Set up and solve systems of equations involving mixtures and percents.
- Set up and solve systems of equations involving costs, perimeters and areas, and other applications.

Distance, Rate, and Time

Example. Elsa's boat has a top speed of 6 miles per hour in still water. While traveling on a river at top speed, she went 10 miles upstream in the same amount of time she went 30 miles downstream. Find the rate of the river current.

Mixtures

Example. Household bleach contains 6% sodium hypochlorite. How much household bleach should be combined with 70 liters of a weaker 1% sodium hypochlorite solution to form a solution that is 2.5% sodium hypochlorite?

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Example. Two factory plants are making TV panels. Yesterday, Plant A produced 7000 fewer panels than Plant B did. Five percent of the panels from Plant A and 3% of the panels from Plant B were defective. If the two plants together produced 1090 defective panels, how many panels did Plant A produce?

Example. A chef plans to mix 100% vinegar with Italian dressing. The Italian dressing contains 12% vinegar. The chef wants to make 160 milliliters of a mixture that contains 23% vinegar. How much vinegar and how much Italian dressing should she use?

Example. Hannah and Linda and Caroline participated in a triathalon relay. Hannah did the swimming, Linda did the biking, and Caroline did the running. The biking distance plus the running distance together was 11 miles. Linda spent 30 minutes on the bike and Caroline spent 15 minutes running. If LindaÕs average speed on the bicycle was 10 mph faster than CarolineÕs average speed running, find LindaÕs average biking speed and CarolineÕs average running speed.

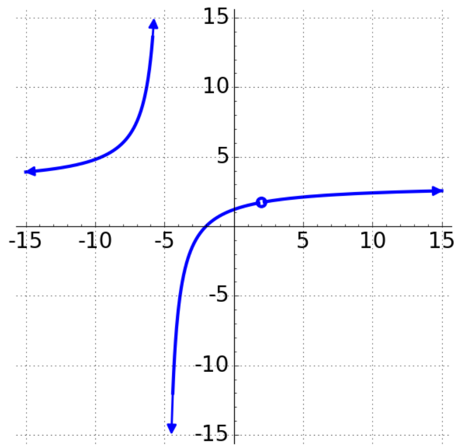
Extra Example. Two trains leave towns 462 miles apart at the same time and travel toward each other. One train travels 20 miles per hour faster than the other. If they meet in 3 hours, what is the rate of each train?

Rational Functions

After completing this section, students should be able to:

- Find the vertical asymptotes of a rational function.
- Find the horizontal asymptotes of a rational function.
- Find the holes of a rational function.
- Find the x- and y-intercepts of a rational function,
- Match equations of rational functions to graphs.
- Write down the equation of a rational function from its graph.

Example. The graph of the function $h(x) = \dfrac{3x^2 - 12}{x^2 + 3x - 10}$ is shown below.



How is the graph of this function $h(x)$ different from the graph of a polynomial?

What is the end behavior of the graph?

What is the behavior of the graph of this function $h(x)$ near $x = -5$?

What is going on at $x = 2$?

For a rational function

- find the vertical asymptotes by:

- find the holes by:

- find the horizontal asymptotes by:

Example. What are the horizontal asymptotes for these functions?

1. $f(x) = \dfrac{5x+4}{3x^2+5x-7}$

2. $g(x) = \dfrac{2x^3+4}{3x^3-7x}$

3. $h(x) = \dfrac{x^2+4x-5}{2x-1}$

Example. Find the vertical asymptotes, horizontal asymptotes, and holes:

$$q(x) = \frac{3x^2 + 3x}{2x^3 + 5x^2 - 3x}$$

END OF VIDEO

Example. Find the horizontal asymptotes of these three functions:

1. $f(x) = \dfrac{-2x^4 - 3x^3 + x^2 + 7x + 1}{5x^4 + 7}$

2. $g(x) = \dfrac{x^2 - 4x + 4}{5x^3 + 9x^2 + 2x - 7}$

3. $h(x) = \dfrac{4x^3 + 5x - 7}{3x^2 - 2x + 6}$

Example. Which of these rational functions has a horizontal asymptote at $y = 2$?

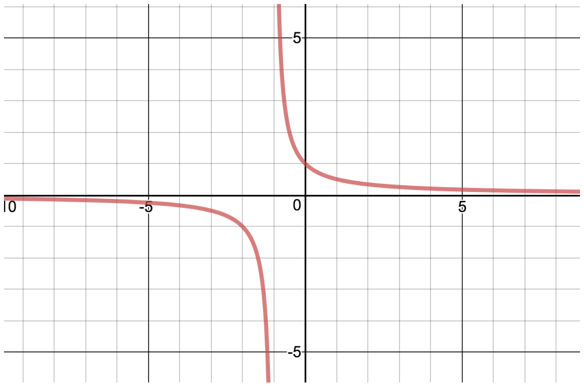
A. $y = \dfrac{2x}{(x-3)(x+5)}$

B. $y = \dfrac{2x^2}{(x-3)(x+5)}$

C. $y = \dfrac{x-2}{2x^2}$

D. $y = \dfrac{(x+4)(x-1)}{(x-2)(x+3)}$

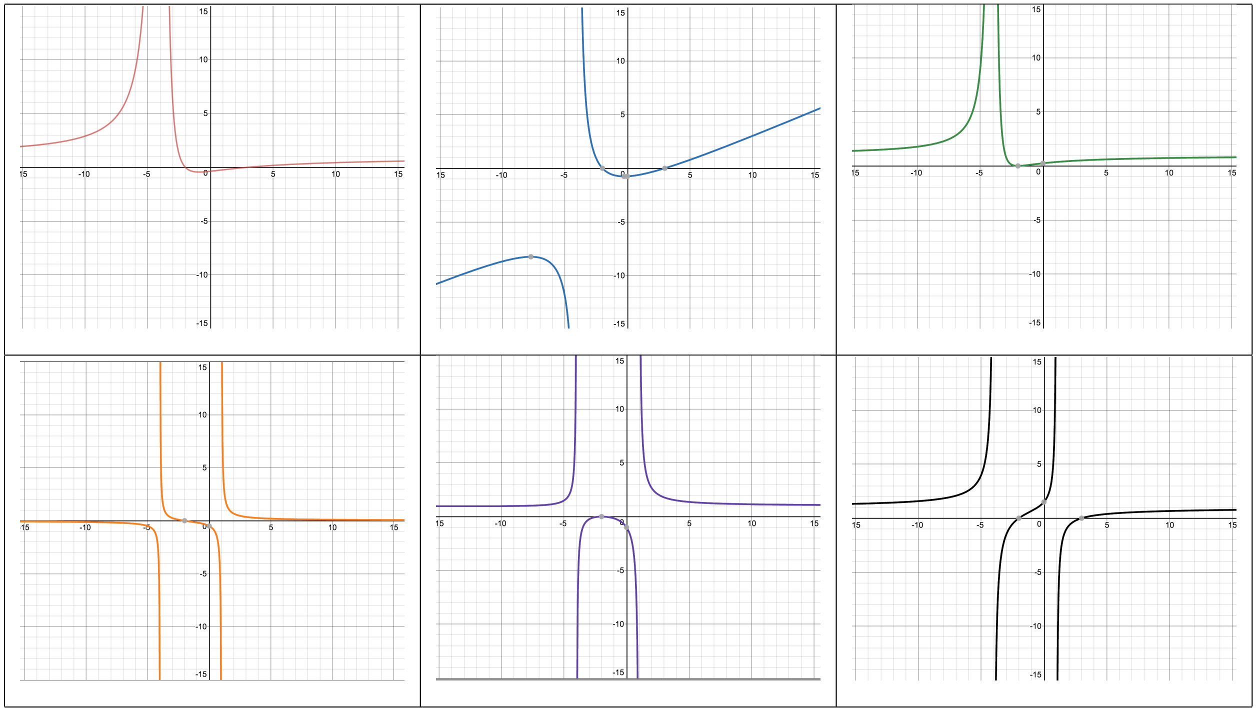
Example. The function $f(x) = \dfrac{x+2}{x^2+3x+2}$ has a vertical asymptote at $x = -1$ and a hole at $x = -2$. How could we predict this from its equation?



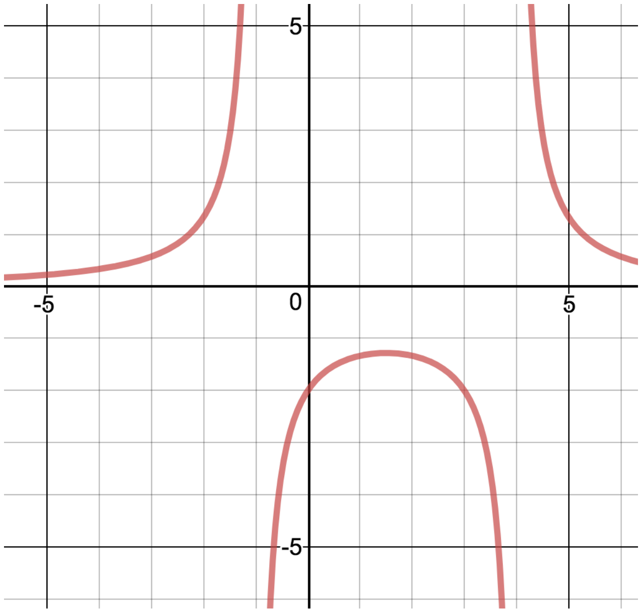
Example. Find the horizontal asymptotes, vertical asymptotes, x- and y-intercepts, and holes of

$$f(x) = \frac{(x-2)(x-3)}{(x-3)(x+1)^2}$$

Example. Find the graph of the function $y = \dfrac{(x-3)(x+2)}{(x+4)(x-1)}$



Example. Find the equation of this rational function.



Combining Functions

After completing this section, students should be able to:

- Find equations for the sum, difference, product, and quotient of two functions, given their equations.
- Use graphs or tables of values to find the sum, difference product, or quotient of two functions, evaluated at a number.

We can combine functions, such as $f(x) = x + 1$ and $g(x) = x^2$ in the following ways.

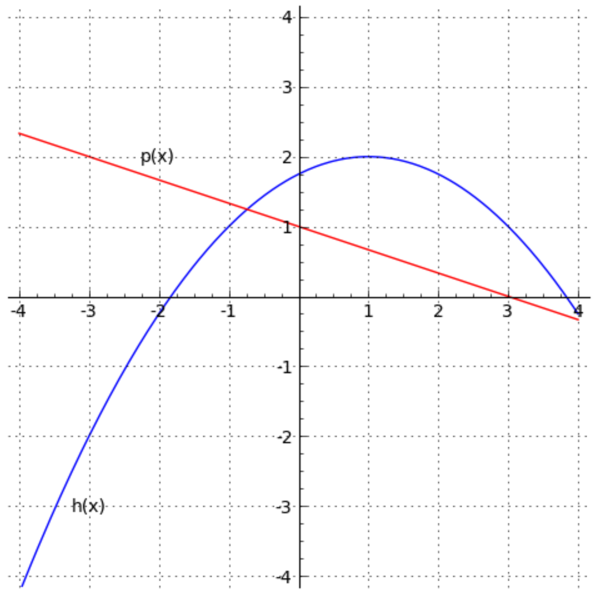
- Add them together: $(f + g)(x) =$

- Subtract them: $(f - g)(x) =$

- Multiply them: $(f \cdot g)(x) =$

- Divide them: $\left(\frac{f}{g}\right)(x) =$

Example. The following graphs represent two functions h and p.



Find

a) $(h - p)(0)$

b) $(ph)(-3)$

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Example. Suppose $f(x) = x^2$ and $g(x) = 3x - 2$. Find

(a) $(f + g)(2)$

(b) $(f - g)(x)$

(c) $(fg)(-1)$

(d) $\frac{f}{g}(x)$

Example. Let t represent the number of years since 2010 and consider the following:

- $f(t)$ is the number of tickets sold per week by Mission X Escape at time t.
- $g(t)$ is the number of tickets sold per week by Bull City Escape at time t.
- $p(t)$ is the cost of a ticket at Mission X as a function of time t.
- $q(t)$ is the cost of a ticket at Bull City Escape as a function of time t.
- $d(t)$ is the population of Durham in thousands of people at time t.

How could you represent the following quantities as functions of t?

1. The total number of escape room tickets sold per week in Durham as a function of time? (Assume there are only the two venues: Mission X and Bull City.)

2. The weekly revenue at of Mission X Escape as a function of time?

3. The total weekly revenue from escape room tickets in Durham, as a function of time?

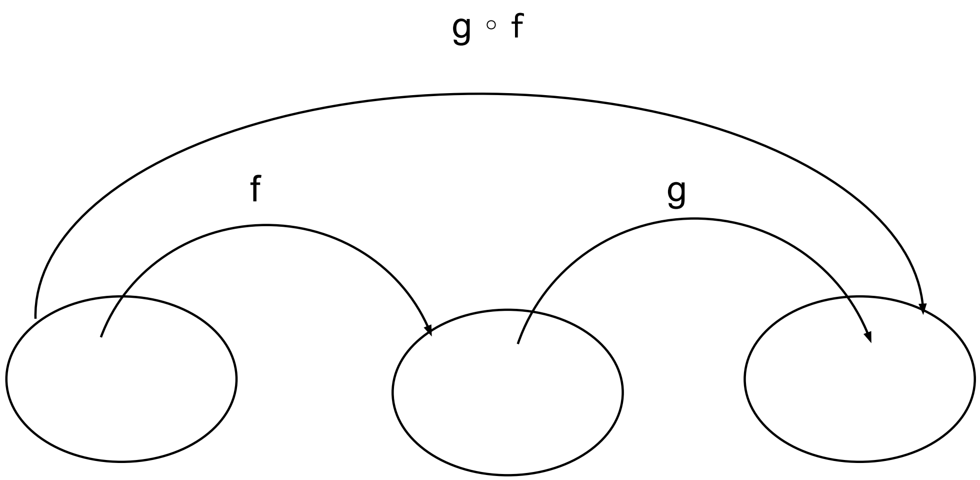
4. The total per capita weekly revenue from escape room tickets in Durham, as a function of time?

Composition of Functions

After completing this section, students should be able to:

- Find the composition of two functions given as equations.
- Use graphs or tables of values to find the composition of two functions, evaluated at a point.
- Decompose a function into the composition of two other functions.

The composition of two functions: $g \circ f(x)$ is defined by:



Example. The tables below define the functions f and g.

x	1	2	3	4	5
$f(x)$	8	3	6	7	4

x	4	5	6	7	8	9
$g(x)$	1	3	8	10	2	2

Find:

a) $g \circ f(4)$

b) $f \circ g(4)$

c) $f \circ f(2)$

d) $f \circ g(6)$

Example. Let $p(x) = x^2 + x$. Let $q(x) = -2x$. Find:

a) $q \circ p(1)$

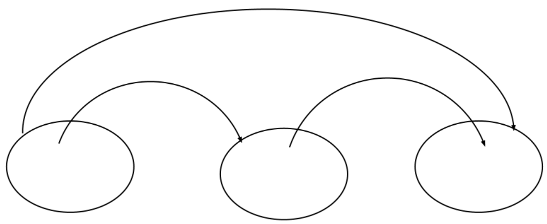
b) $q \circ p(x)$

c) $p \circ q(x)$

d) $p \circ p(x)$

Note. In general, $f \circ g \neq g \circ f$!

Example. $h(x) = \sqrt{x^2 + 7}$. Find functions f and g so that $h(x) = f \circ g(x)$.



Example. $r(x) = (7x + 2)^3$. Find f and g such that $r(x) = f \circ g(x)$.

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Example. Two children decided to run a lemonade and hot chocolate stand. The function

$$f(x) = 3|x - 65| + 10$$

represents the number of drinks sold as a function of the day's high temperature in degrees Fahrenheit. The function

$$g(x) = 0.5x - 5$$

represents the profit in dollars as a function of the number of drinks sold.

The kids want to know how much they'll make if the day's high temperature is 75° degrees. Find the answer for them.

In general, how much money will they make as a function of the temperature in degrees Fahrneheit?

Example. Let $f(x) = \frac{1}{x}$ and $g(x) = x^2 + 3x$. Find

a) $f \circ g(x)$

b) $g \circ f(x)$

c) $g \circ g(x)$

d) $f \circ f(x)$

Example. Consider the function $H(x) = \sqrt[3]{\frac{1}{x+1}}$. Find two function f and g such that $H(x) = f \circ g(x)$.

Example. Consider the function $P(x) = 4e^{3x-7}$. Find two function f and g such that $H(x) = f \circ g(x)$.

Inverse functions

After completing this section, students should be able to:

- Based on the graph of a function, determine if the function has an inverse that is a function.
- Draw the graph of an inverse function, given the graph of the original.
- Use a table of values for a function to write a table of values for its inverse.
- Determine if two given functions are inverses of each other by computing their compositions.
- Use a formula for a function to find a formula for its inverse.
- Find the range of the inverse function from the domain of the original function.
- Find the domain of the inverse function from the range of the original function.

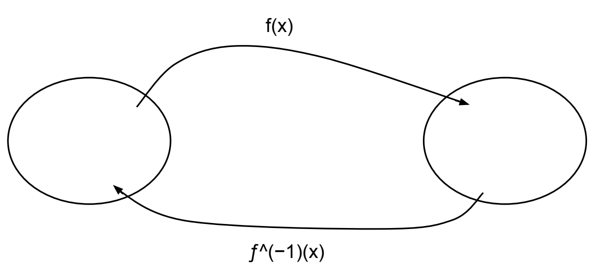
Example. Suppose $f(x)$ is the function defined by the chart below:

x	2	3	4	5
$f(x)$	3	5	6	1

In other words,

- $f(2) = 3$
- $f(3) = 5$
- $f(4) = 6$
- $f(5) = 1$

Definition. The **inverse function** for f, written $f^{-1}(x)$, undoes what f does.

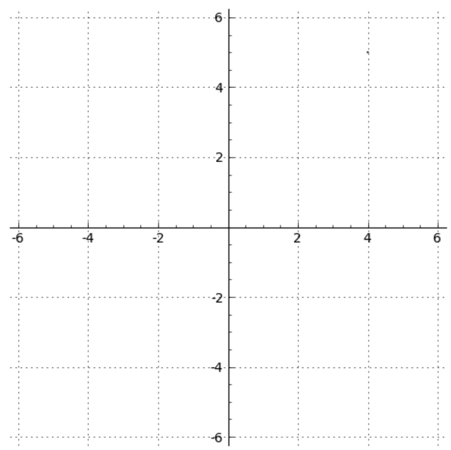


- $f^{-1}(\ 3\ ) = 2$
- $f^{-1}(\quad) =$
- $f^{-1}(\quad) =$
- $f^{-1}(\quad) =$

x	3			
$f^{-1}(x)$	2			

Key Fact 1. Inverse functions reverse the roles of y and x.

Graph $y = f(x)$ and $y = f^{-1}(x)$ on the same axes below. What do you notice about the points on the graph of $y = f(x)$ and the points on the graph of $y = f^{-1}$?



Key Fact 2. The graph of $y = f^{-1}(x)$ is obtained from the graph of $y = f(x)$ by reflecting over the line ________ .

In our same example, compute:

$f^{-1} \circ f(2) =$	$f \circ f^{-1}(3) =$
$f^{-1} \circ f(3) =$	$f \circ f^{-1}(5) =$
$f^{-1} \circ f(4) =$	$f \circ f^{-1}(6) =$
$f^{-1} \circ f(5) =$	$f \circ f^{-1}(1) =$

Key Fact 3. $f^{-1} \circ f(x) =$ ________ and $f \circ f^{-1}(x) =$ ________ . This is the mathematical way of saying that f and f^{-1} undo each other.

Example. $f(x) = x^3$. Guess what the inverse of f should be. Remember, f^{-1} undoes the work that f does.

Example. Find the inverse of the function:

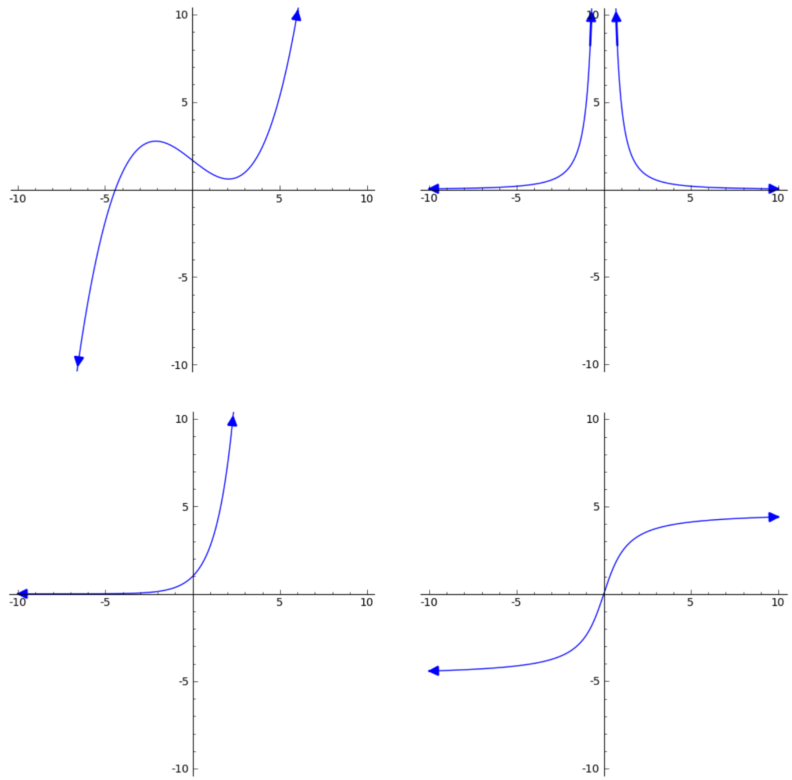
$$f(x) = \frac{5-x}{3x}$$

Note. $f^{-1}(x)$ means the inverse function for $f(x)$. Note that $f^{-1}(x) \neq \frac{1}{f(x)}$.

Question. Do all functions have inverse functions? That is, for any function that you might encounter, is there always a **function** that is its inverse?

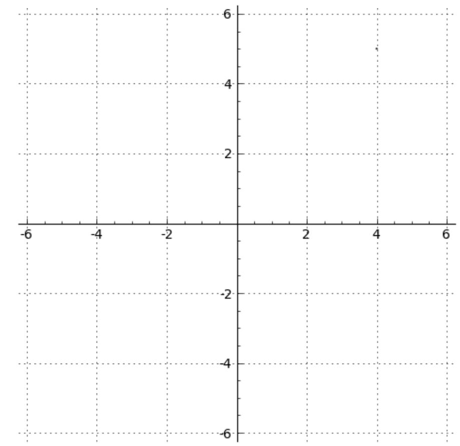
Try to find an example of a function that does **not** have an inverse **function**.

Key Fact 4. A function f has an inverse function if and only if the graph of f satisfies the **horizontal line test** (i.e. every horizontal line intersects the graph of $y = f(x)$ in at most one point.)



Definition. A function is **one-to-one** if it passes the horizontal line test. Equivalently, a function is one-to-one if for any two different x-values x_1 and x_2, $f(x_1)$ and $f(x_2)$ are different numbers. Sometimes, this is said: f is one-to-one if, whenever $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Example. (Tricky) Find $p^{-1}(x)$, where $p(x) = \sqrt{x-2}$ drawn above. Graph $p^{-1}(x)$ on the same axes as $p(x)$.



For the function $p(x) = \sqrt{x-2}$, what is:

- the domain of p?
- the range of p?
- the domain of p^{-1}?
- the range of p^{-1} ?

Key Fact 5. For any invertible function f, the domain of $f^{-1}(x)$ is ______________ and the range of $f^{-1}(x)$ is ______________ .

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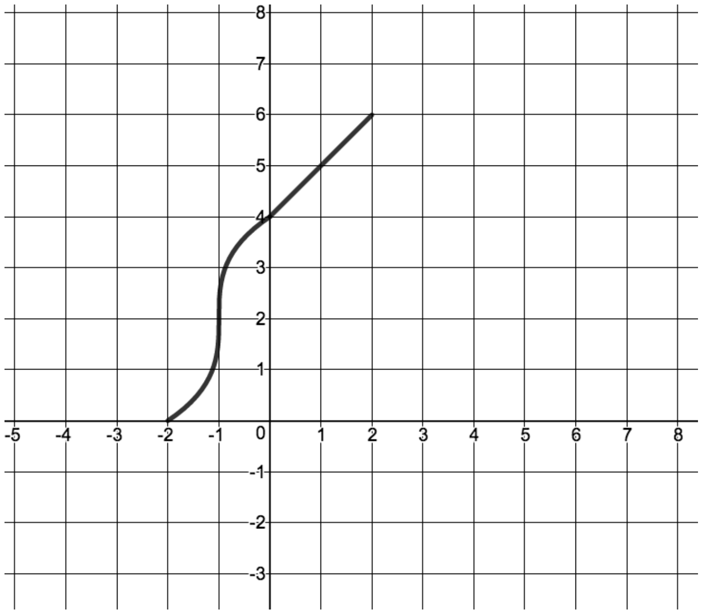
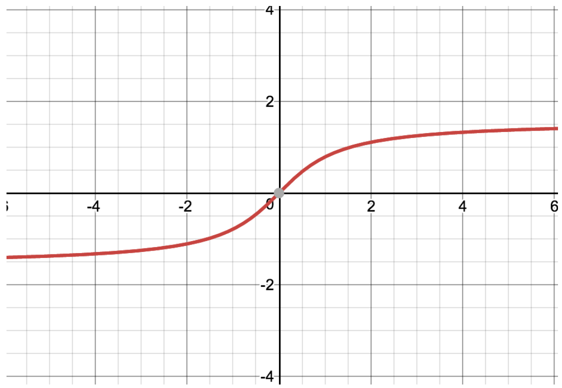
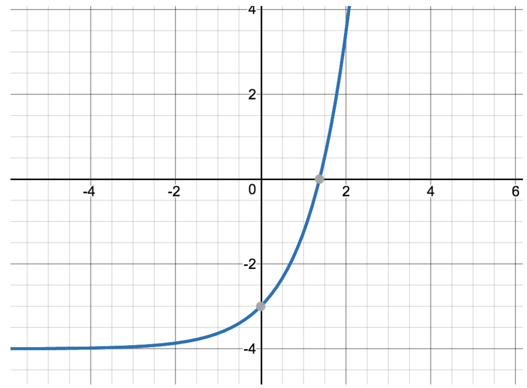
What are some facts about inverse functions?

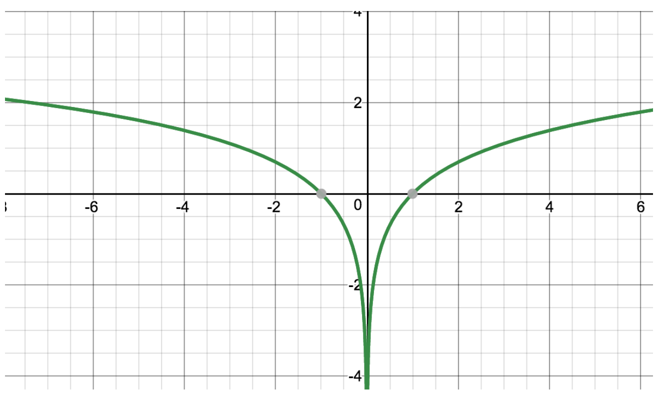
Example. The graph of $f(x)$ is show below. Find the graph of $f^{-1}(x)$.

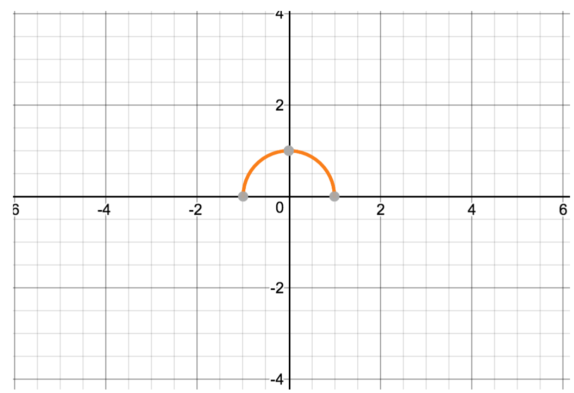


Example. For each function graph, determine whether it has an inverse function.

A. 

B. 

C. 

D. 

Example. Consider the function $g(x) = \dfrac{5-x}{x+2}$. What is the RANGE of $g^{-1}(x)$?
Hint: you do not need to compute a formula for $g^{-1}(x)$ to answer this question.

Example. If $f(x) = \frac{x}{2} - 1$, what is the inverse function $f^{-1}(x)$?

A) $p(x) = \dfrac{1}{\frac{x}{2} - 1}$

B) $r(x) = \dfrac{2}{x} + 1$

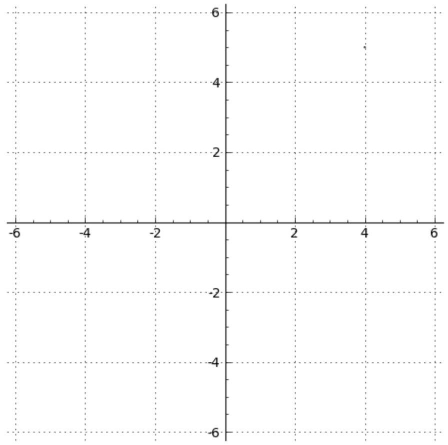
C) $v(x) = 2(x + 1)$

D) $w(x) = 2x + 1$

Example. Find the inverse function for the exponential function $f(x) = 10^x$.

Example. Find the inverse function for the exponential function $g(x) = e^x$.

Example. Graph $g(x) = 10^x$ and its inverse $g^{-1}(x) = \log_{10} x$ on the same axes.



List the properties of the graphs, including domain, range, and asymptotes.

$y = 10^x$

$y = \log_{10}(x)$

Example. Find the inverse of the function:

$$f(x) = \frac{3x + 1}{x - 6}$$

Example. Find the inverse of the function: $f(x) = \dfrac{7-x}{2x+3}$

Find the domain and range of $f(x)$ and $f^{-1}(x)$.

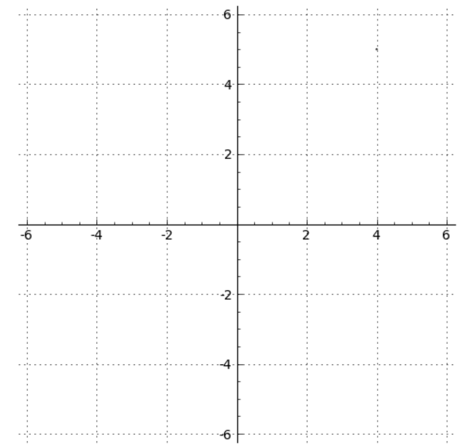
Example. According to math lore, if you are age x, the oldest person that it is okay for you to date is given by the formula $d(x) = 2x - 14$. Plug in your own age for x and see how old a person you can date.

Suppose you want to date a younger person instead of an older person. Invert the formula to find out how young a person someone of a given age can date.

Plug in your own age for x into $d^{-1}(x)$ and see how young a person you can date.

Extra Example. $h(x) = 7 - x^3$. Find $h^{-1}(x)$ by reversing the roles of y and x and solving for y.

Extra Example. Find $f^{-1}(x)$, where $f(x) = \sqrt{x+1}$. Graph $f^{-1}(x)$ on the same axes as $f(x)$.



For the function $f(x) = \sqrt{x+1}$, what is:

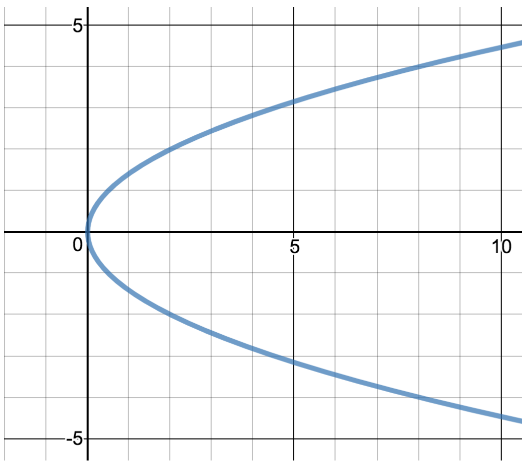
- the domain of f?
- the range of f?
- the domain of f^{-1}?
- the range of f^{-1} ?

Symmetry and Even and Odd Functions

After completing this sections, students should be able to:

- Identify whether a graph is symmetric with respect to the x-axis, symmetric with respect to the y-axis, symmetric with respect to the origin, or none of these.
- Determine whether a function is even or odd or neither, based in its equation.

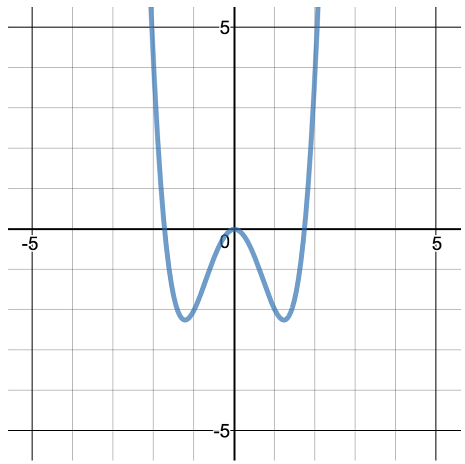
Definition. A graph is *symmetric with respect to the x-axis* if ...



Whenever a point (x, y) is on the graph, the point

is also on the graph.

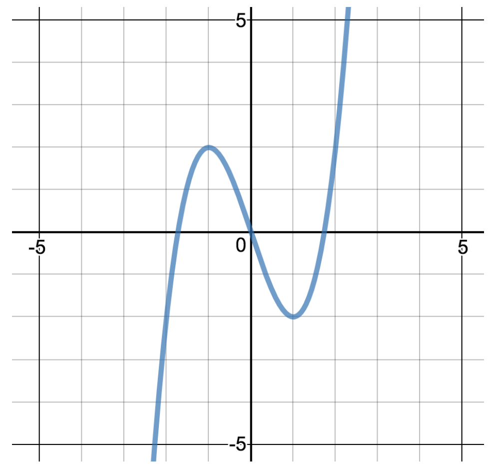
Definition. A graph is *symmetric with respect to the y-axis* if ...



Whenever a point (x, y) is on the graph, the point

is also on the graph.

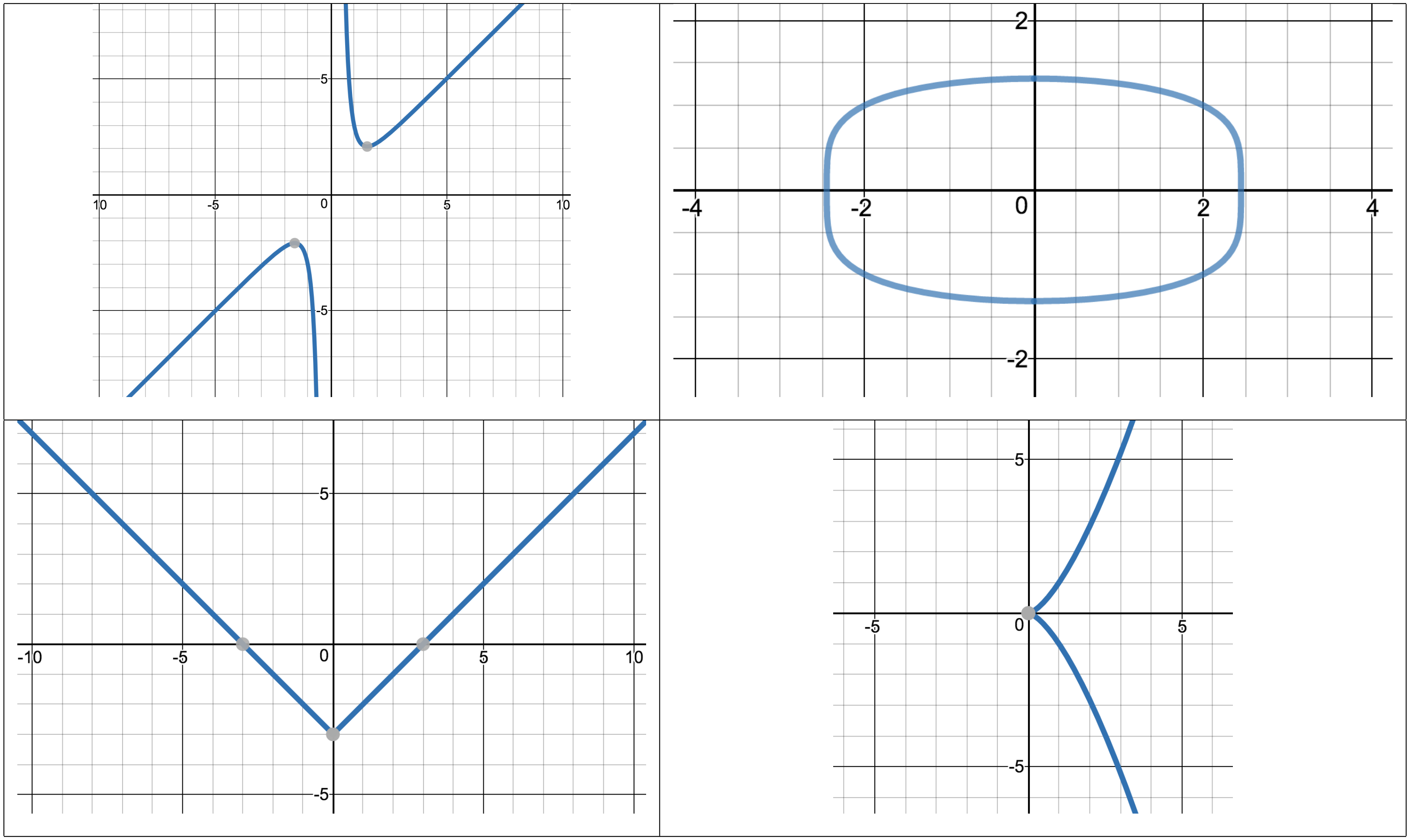
Definition. A graph is *symmetric with respect to the origin* if ...



Whenever a point (x, y) is on the graph, the point

is also on the graph.

Example. Which graphs are symmetric with respect to the x-axis, the y-axis, the origin, or neither?



Example. Which graphs are symmetric with respect to the x-axis, the y-axis, the origin, or neither?

1. $y = \dfrac{2}{x^3} + x$

2. $x^2 + 2y^4 = 6$

Definition. A function $f(x)$ is *even* if ...

Example. $f(x) = x^2 + 3$ is even because ...

Definition. A function $f(x)$ is *odd* if ...

Example. $f(x) = 5x - \frac{1}{x}$ is odd because ...

Question. There is no word like even or odd for when a function's graph is symmetric with respect to the x-axis. Why not?

Example. Determine whether the functions are even, odd, or neither.

1. $f(x) = 4x^3 + 2x$

2. $g(x) = 5x^4 - 3x^2 + 1$

3. $h(x) = 2x^3 + 7x^2$

Piecewise Functions

After completing this sections, students should be able to:

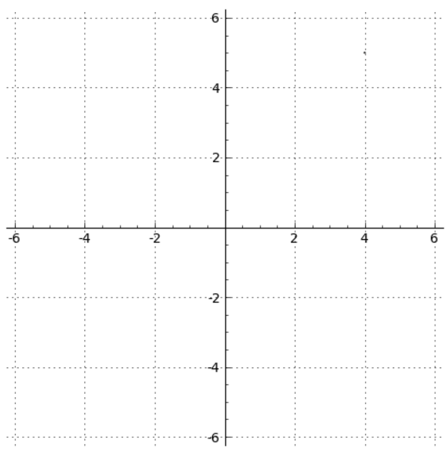
- Evaluate piecewise functions at a given x-value.
- Graph piecewise functions

Example. The function f is defined as follows:

$$f(x) = \begin{cases} 1 - x^2 & \text{if } x < 1 \\ -2x + 3 & \text{if } x \geq 1 \end{cases}$$

1. What is $f(-2)$? What is $f(1)$?

2. Graph $y = f(x)$.



3. Is $f(x)$ continuous?

Another example, fill in from notes

Review

Decide which of the following statements are true. (Select all correct answers, and assume all arguments of ln are positive numbers.)

A. $\sqrt{x^2+4} = x+2$

B. $\log(\frac{1}{a}) = -\log(a)$

C. $10^{2\log x} = x^2$

D. $\log(w-v) = \frac{\log(w)}{\log(v)}$

E. $\log_2(x+8) = \log_2(x)+3$

What are some types of equations we have solved?

What are some techniques for solving equations?

What are extraneous solutions? When do we have to watch for them?

What types of equations are these? What technique is needed to solve them?

1. $\dfrac{3}{2-x} = \dfrac{4}{x}$
2. $(x-2)(x+1) = 3$ for x
3. $2^x = 3^{x+1}$
4. $5x^5 = 17$
5. $7\log(5x) = 2$
6. $\sqrt{4+x} + 2 = x$
7. $3x^{2/3} + 1 = 28$
8. $y = \dfrac{4-3x}{5x-9}$ for x

What kinds of inequalities have we solved? What techniques do we use?

1. $\dfrac{x^2 - 1}{x + 3} > 0$

2. $7 < 1 - 3|2 - 4x|$

3. $x^3 + 5x^2 \leq 6x$

4. $8x + 4 > 5x - 2$ and $2 - 3x < 5x$

What do we have to worry about when looking for domains?

What types of equations are these? What technique is needed to solve them?

1. $A = P(1 + rt)$ for r
2. $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ for b
3. $y = \frac{4 - 3x}{5x - 9}$ for x
4. $a + b = rx - 4x^2$ for x
5. $M = \log(\frac{I}{S})$ for S
6. $ba^t = 3 + x$ for t
7. $ba^t = 3 + x$ for a
8. $\sqrt{4 + x} = 3y$ for x