Part XVI

Topology

This section is an introduction to the topology of surfaces, which we will need to prove the Magic Theorem.

Some of this material is taken from *The Shape of Space* by Jeff Weeks.

Download Torus Games from http://www.geometrygames.org/TorusGames/index.html for from Google Play to play Tic-Tac-Toe on the torus and other games.

What is topology?

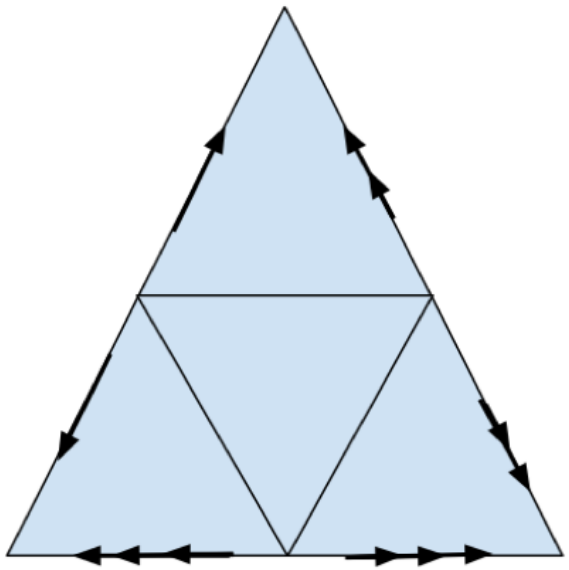
- The properties of an object that change when you bend or stretch it are the geometry of the object.
  - For example, angles, distances, and curvature are parts of geometry but not topology.
- The properties of an object that stay the same when you bend or stretch it are called the topology of the object.
  - Two objects are considered the same topologically if you can deform one into the other without tearing, cutting, fusing, or other violent actions.

Deforming an object doesn't change its topology

Which surfaces are topologically the same?

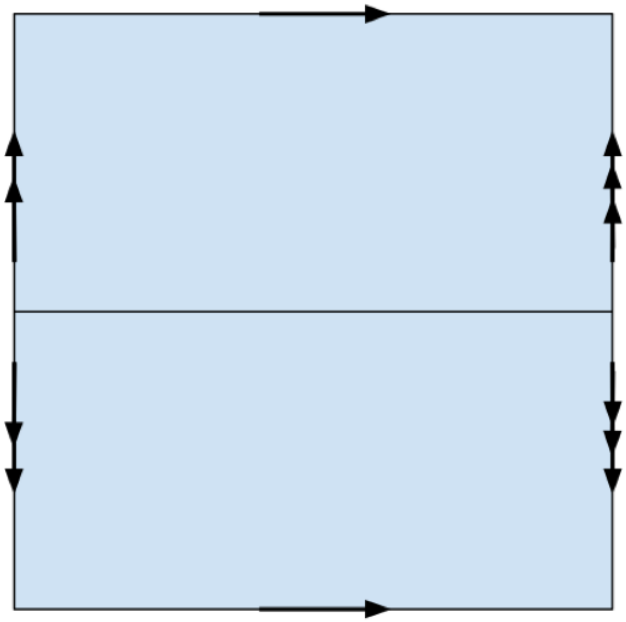
Gluing diagrams

What topological surface do you get when you glue (or tape) the edges of the triangle together as shown?



Another gluing diagram

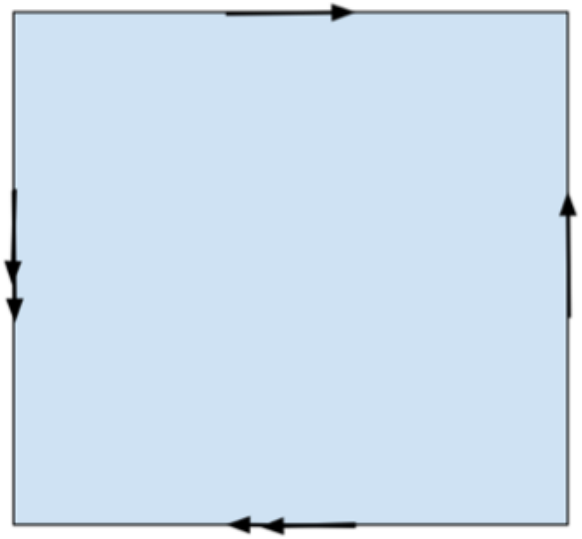
What do you get when you glue the edges of the square together like this?



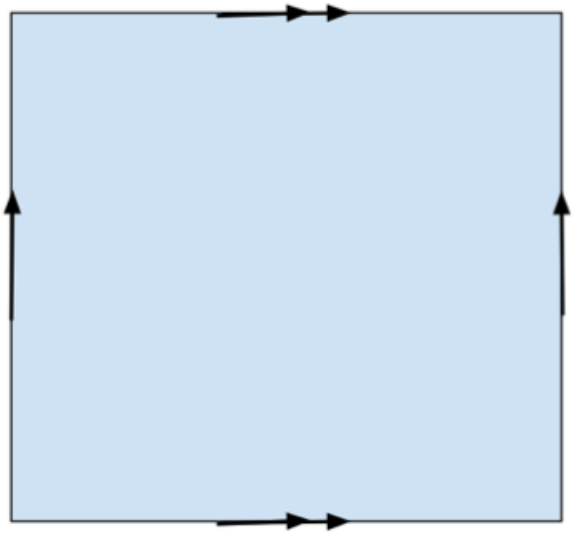
Dont glue the interior parts of the square together, just the edges!

More gluing diagrams

- What surface is this?

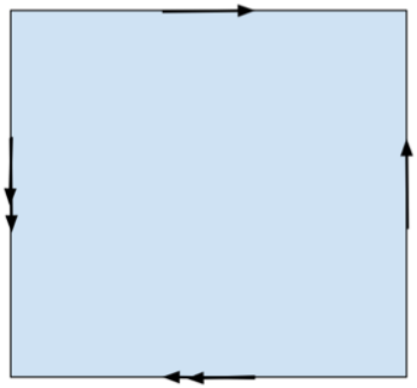


- And this?



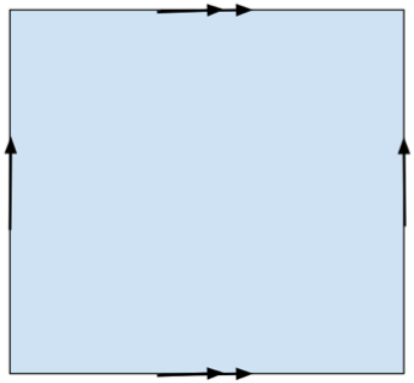
Gluing diagrams answers

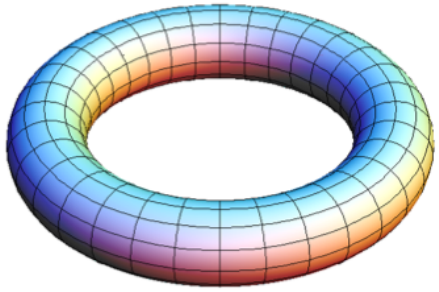
- What surface is this?



S^2 (a sphere)

- And this?

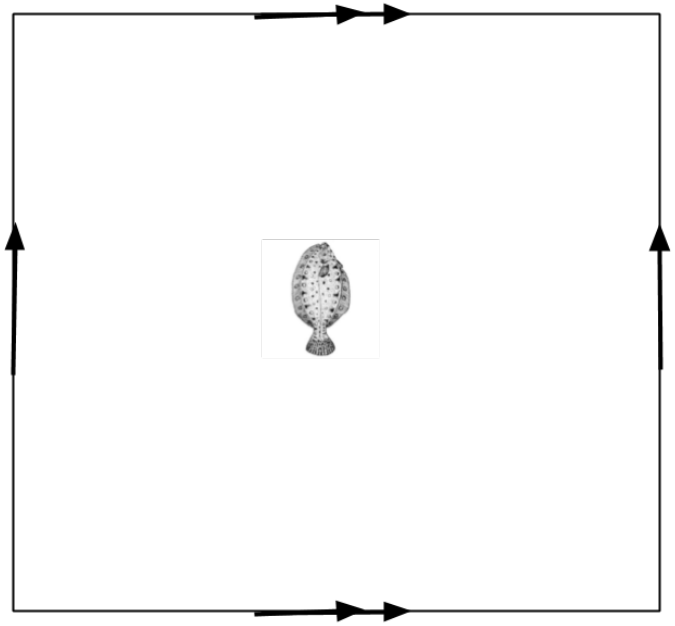




T^2 (a torus)

Life in a torus

What happens as this 2-dimensional creature travels through its tiny universe?

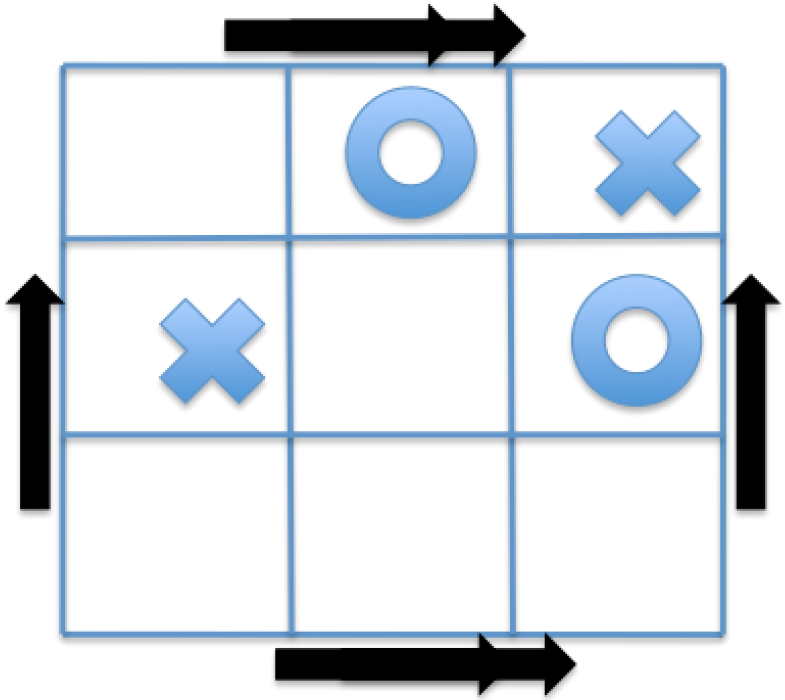


What does it see when it looks forwards? Backwards? Left? Right?

See Torus Games animation

Tic-Tac-Toe on a Torus

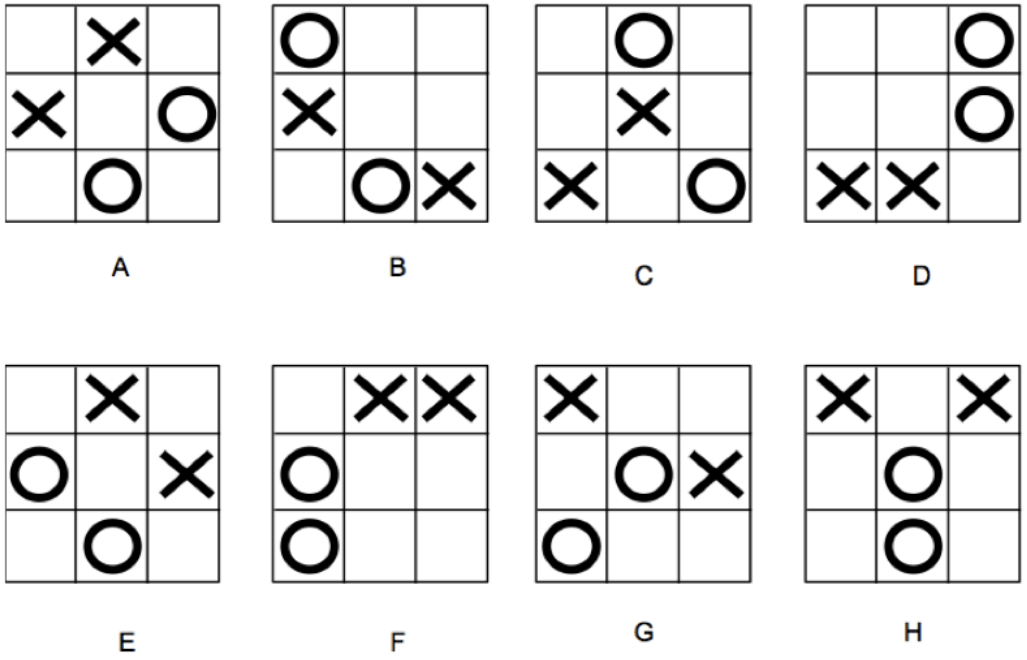
Where should X go to win?



What if it's O's turn?

Equivalent positions in torus tic-tac-toe

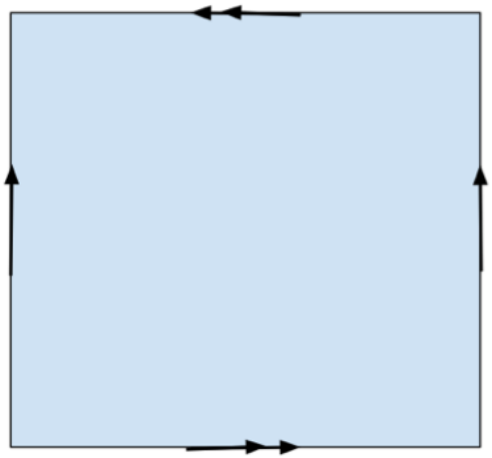
Which of the following positions are equivalent in torus tic-tac-toe?



From *The Shape of Space* by Jeff Weeks.

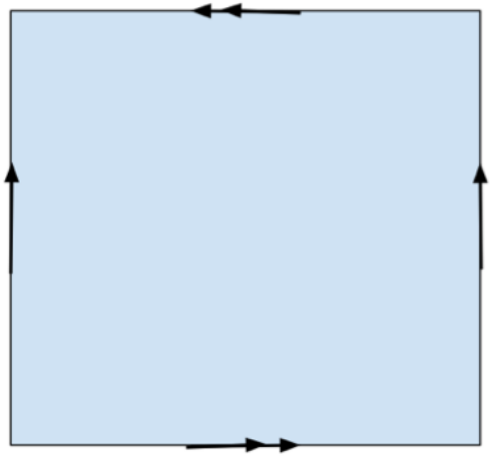
Another surface

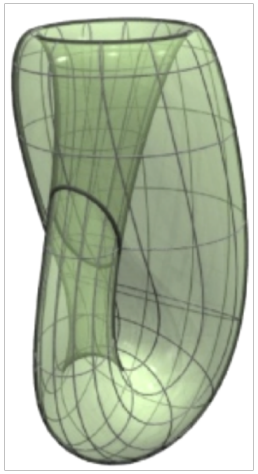
What surface do you get when you glue together the sides of the square as shown?



Another surface answer

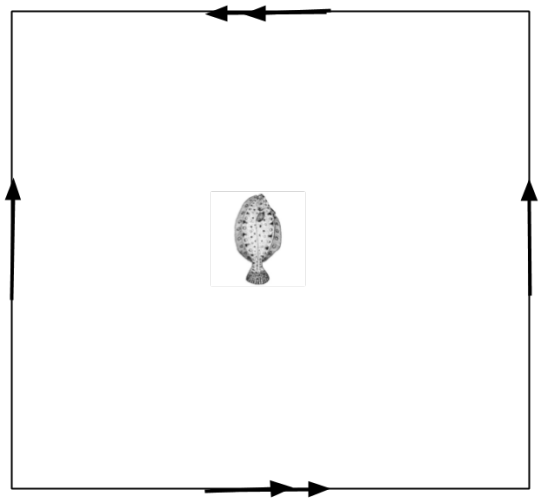
What surface do you get when you glue together the sides of the square as shown?





Life in a Klein bottle surface

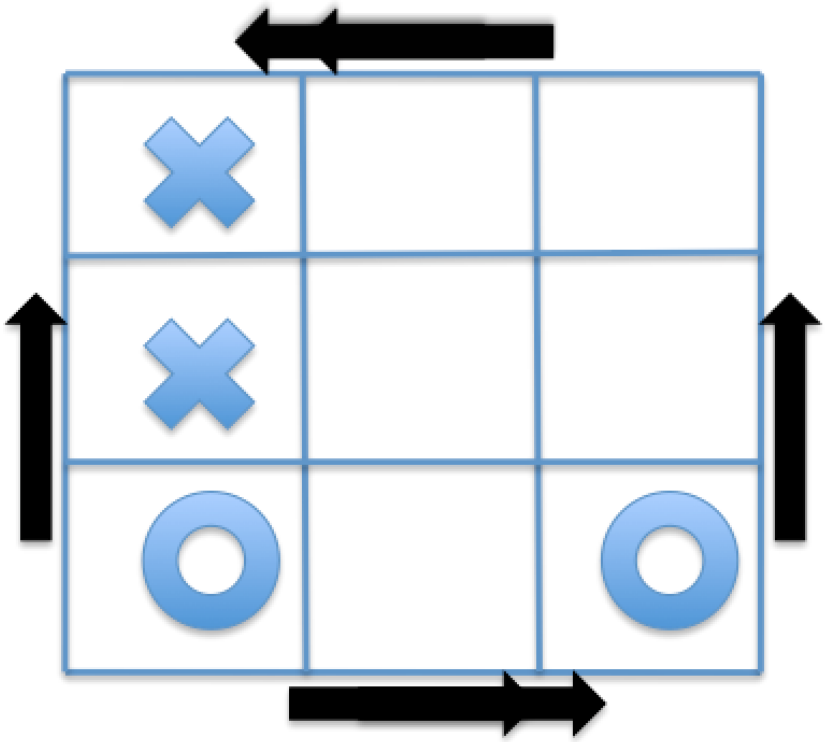
What happens as this creature travels through its Klein bottle universe?



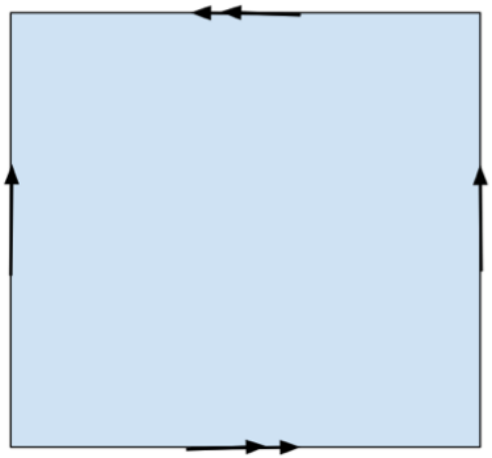
- A path that brings a traveler back to his starting point mirror-reversed is called an orientation-reversing path.
- A surface that contains an orientation-reversing path is called non-orientable.

Tic Tac Toe on a Klein Bottle

Where should X go to win?



What happens when you cut a Klein bottle in half?



It depends on how you cut it.
Cutting a Klein bottle in half

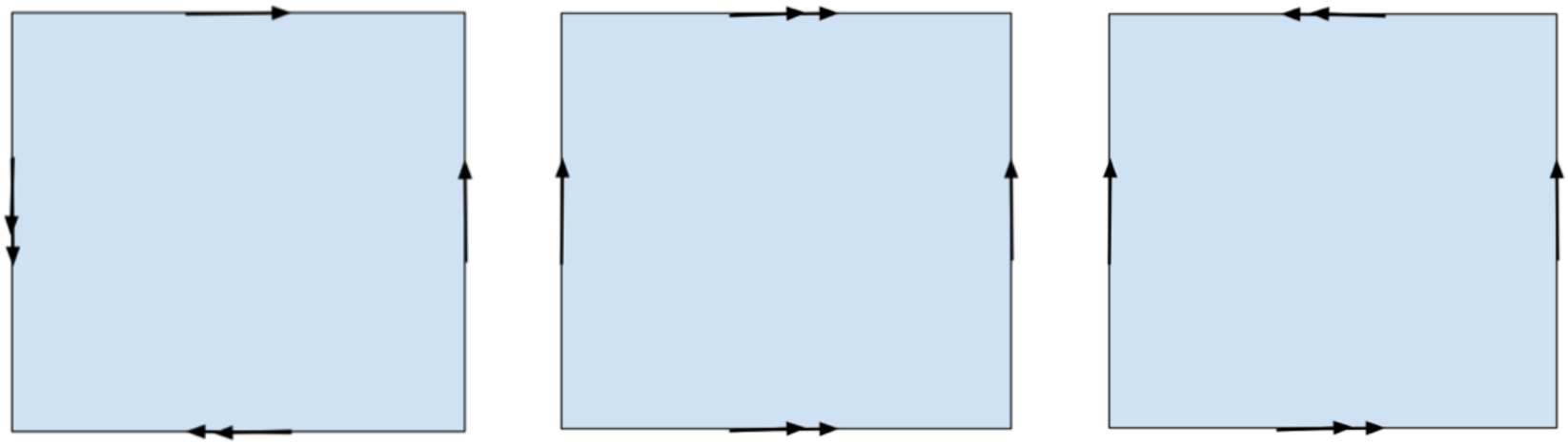
3-dimensional spaces

- How can you make a 3-dimensional space that is analogous to the 2-dimensional torus?

- Is there a 3-dimensional analog to the Klein bottle?

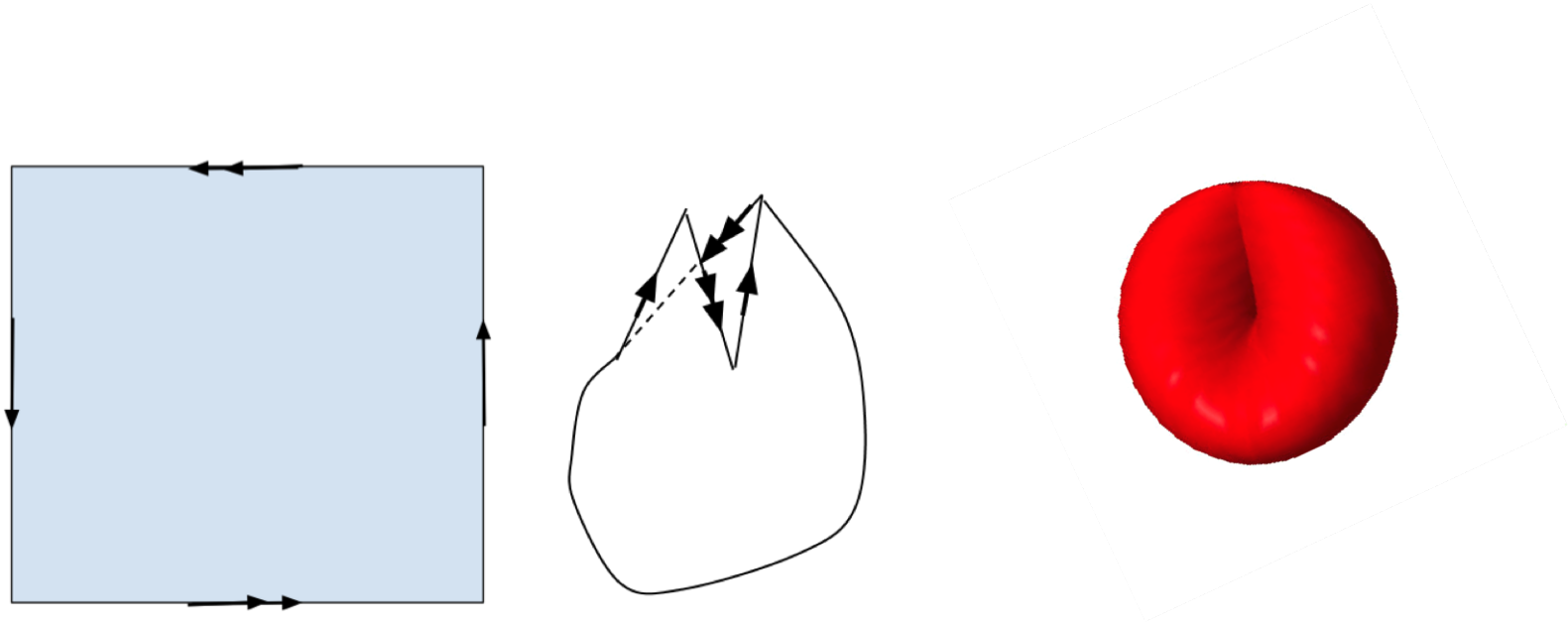
Surfaces made from a square

We have seen that the following gluing diagrams describe a topological sphere, torus, and Klein bottle.



What other topological surfaces result from gluing the edges of a square?

The Projective Plane



Homework

1. Decide which of the tic-tac-toe positions are equivalent and which ones are different. (In other words, divide them into groups based on which ones are equivalent.) Explain how you know for sure that they are not all equivalent. Hint: find some property that does not change when you scroll or rotate that is different between them.
2. Choose one of these two assignments:
   - Find a winning strategy for torus tic-tac-toe and explain it so that anyone can follow it to win (as the first player to move), OR
   - Design a torus or Klein bottle word search like with at least 10 words hidden. Circle the words. Your game should make use of the gluing properties of the torus or Klein bottle.

Word Searches

h	l	m	e	a	i
n	a	l	n	b	r
j	l	d	a	e	a
c	a	t	w	m	t
x	e	g	i	p	a
o	s	s	u	m	p

bat
camel
cat
jaguar
llama
ox
panda
pig
possum
puma
rat
seal
shrew
tapir

h	p	m	a	c	m
a	d	e	c	k	r
a	d	n	f	e	l
a	l	i	l	m	c
s	r	q	r	e	a
h	a	h	o	c	l

ash
birch
cedar
elm
fir
larch

From *Geometry Games* by Jeff Weeks.