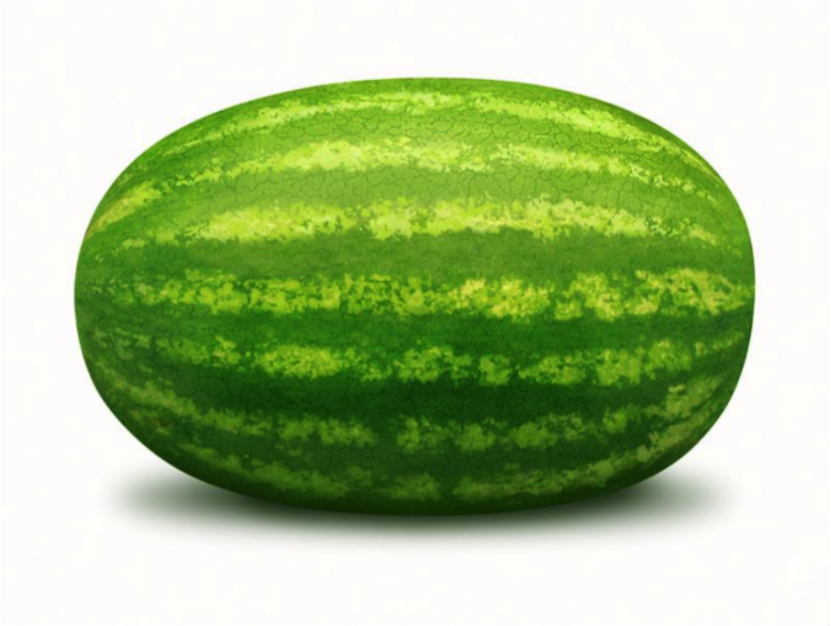
Part XX

Curvature

The goal for this part is to calculate curvature of surfaces.

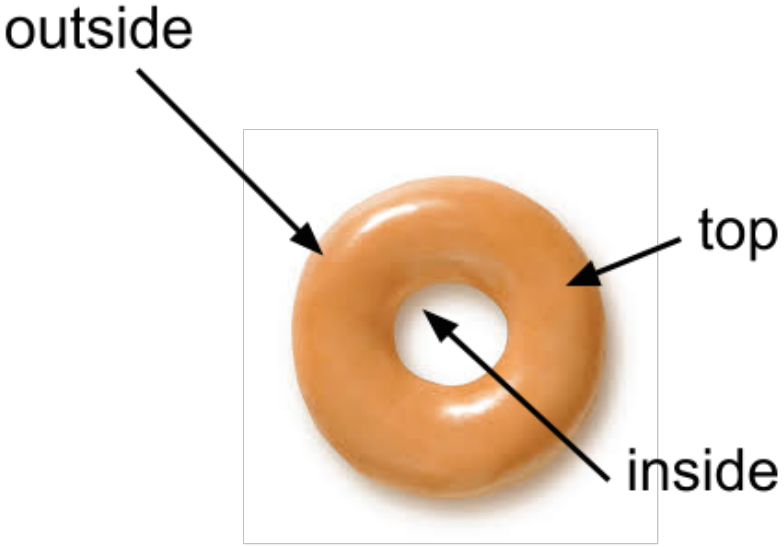
Intuition about curvature

Which would you say is more curved, a piece of the surface of an orange or the same size piece of the surface of a watermelon?



Curvature of donut parts

Examine the surface of a Krispy Kreme donut.



How does the curvature on the inside of the hole compare to the curvature on the outside or on the top?

Flattening curved surfaces

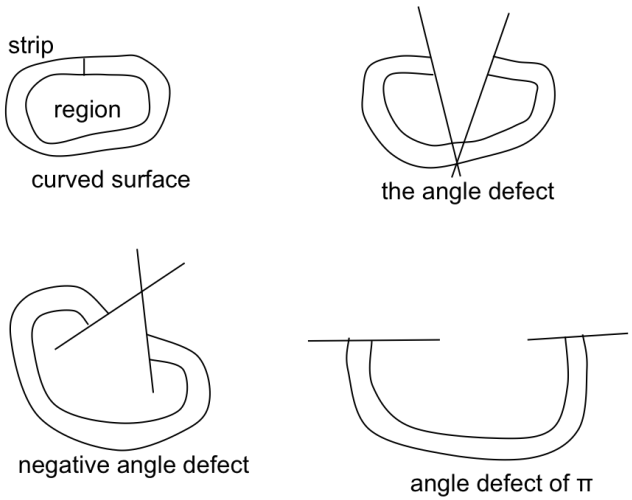
One way you know that a surface is curved is that it doesnt flatten without tearing or stretching.

How can you quantify the curvature in a piece of orange peel?

Angle defect

One way to quantify the amount of curvature inside a region of surface is by calculating angle defect.

1. First, cut out a disk-like region of the surface.
2. Next, cut a thin strip around the boundary of this region.
3. Fatten the strip out as much as possible.
4. Measure the angle between the edges of the strip. This is the curvature inside the region.



If the strip meets up with itself perfectly, then the region has zero angle defect regional curvature. Sometimes, the strip doesnt meet up because it doesnt curl enough. This is positive curvature. Sometimes the strip doesnt meet up because it curls around too much and overshoots. This is negative curvature.

Estimate regional curvature

Find the regional curvature of:

a piece of orange peel

a piece of a kale leaf

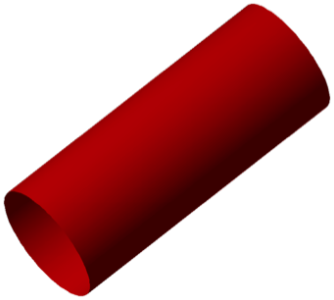
a piece of banana peel

a piece of a cabbage leaf

Regional curvature of paper products

One nice feature about the angle defect regional curvature is that it is possible to measure curvature even at regions of a surface that are not smooth, like the cone points on a cone or vertices on a polyhedron.

Find the angle defect of

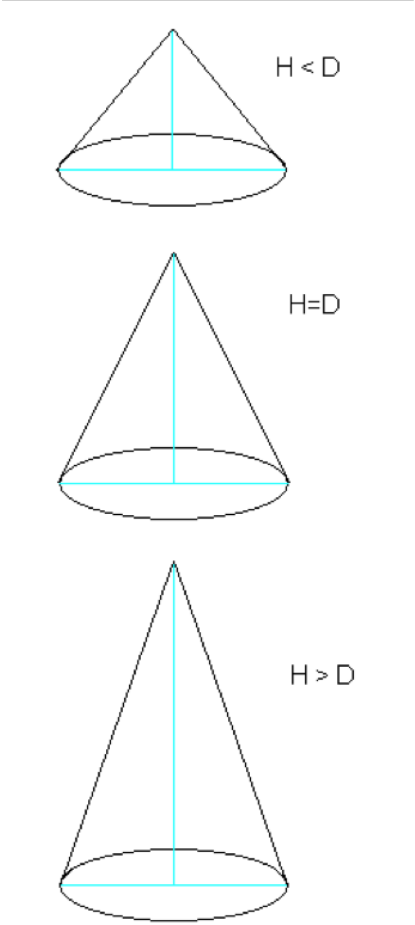


a piece of the side of a cylinder

a piece of a cone
that does not contain the cone point

Curvature at a cone point

Which cone has more angle defect around its cone point?



Flattening a cone