Part XXIV

Spherical Symmetry Patterns

The goal for this part is to identify and classify spherical symmetry patterns.

Symmetry of sports balls

What symmetries do you see in these balls?

What signatures could we give them?

Symmetry of everyday objects

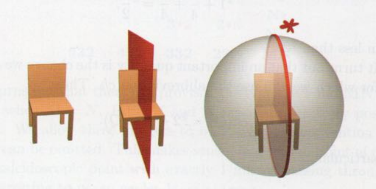
Many other everyday 3-dimensional objects have spherical symmetry.

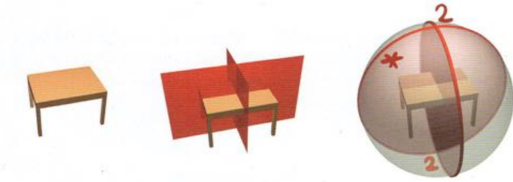
What symmetries do you see in these objects?

What are their signatures?

Symmetry of 3D objects

It can be helpful to think of them as lying inside a sphere.





Identify a spherical symmetry pattern

1. Pick an object.
2. Find its signature.
3. Find its "cost", using the usual cost table.
4. Identify its orbifold.

Symbol	Cost (\$)	Symbol	Cost (\$)
$\circ$	2	$*$ or $\times$	1
2 (for gyration point)	1/2	2 (for mirror point)	1/4
3 (for gyration point)	2/3	3 (for mirror point)	2/6
4 (for gyration point)	3/4	4 (for mirror point)	3/8
5 (for gyration point)	4/5	5 (for mirror point)	4/10
...	...	...	...
N (for gyration point)	$\frac{N-1}{N}$	N (for mirror point)	$\frac{N-1}{2N}$
∞ (for gyration point)		∞ (for mirror point)	

What do you notice about the costs of the spherical symmetry patterns?

The Magic Theorem for spherical symmetry patterns

State a Magic Theorem for spherical symmetry patterns.

Why does this make sense in terms of the Gauss-Bonnet Theorem?

- For a spherical symmetry pattern, the underlying orbifold must have total curvature that is ...

- The orbifold must have Euler characteristic that is ....

Spherical symmetry and the Gauss-Bonnet Theorem

- The orbifold must have the topology of one of the following surfaces ....
- In each case, the curvature of from the cone points and corners must add up to ...
- In each case, the cost of from the cone points and corners must add up to ...
- In each case, the cost of from the cone points and corners plus the $1 for the X or $\star$ must add up to ...

What signatures are possible for spherical symmetry patterns?

Using the Magic Theorem (or the Gauss-Bonnet Theorem), find all possible spherical symmetry patterns. Consider these symmetry patterns separately:

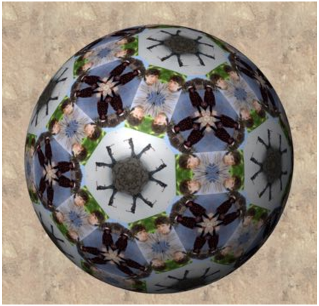
1. symmetry patterns whose orbifold is topologically a sphere.
2. symmetry patterns whose orbifold is topologically a disk.
3. symmetry patterns whose orbifold is topologically a projective plane.

Temari balls

Sphereflakes and Kaleidotile

Watch Vi Hart's video on Sphereflakes.

Experiment with spherical symmetry using Jeff Week's program Kaleidotile.



Homework

1. Read Chapter 4 from the textbook on Spherical Symmetry Patterns.
2. Find the signature of these temari balls.

Give the signature. For the last two, make your best guess for the parts you can't see. Remember that a zig-zag has 180° rotational symmetry.