

Part XXVII

Hyperbolic Symmetry Patterns

This part is an introduction to representations of the hyperbolic plane and hyperbolic wallpaper patterns.

Hyperbolic soccerball symmetry

What is the signature of the hyperbolic soccerball pattern?

What is its cost?

Additional hyperbolic tilings

Are there other shapes that you could attach together to get a hyperbolic (negatively curved) wallpaper pattern?

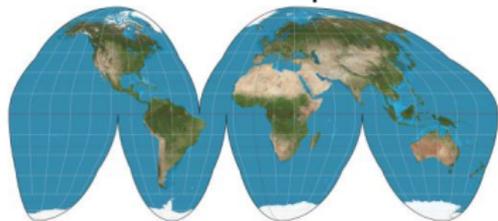
What is true about the cost of a negatively curved wallpaper pattern?

Other representations of positively and negatively curved spaces

- ▶ It is hard to work with curly, floppy hyperbolic surfaces.
- ▶ So it can be helpful to represent the "hyperbolic plane" on flat paper.
- ▶ With positively curved surfaces (like orange peels), it is not possible to flatten the surface without stretching it or tearing it.
- ▶ With negatively curved surfaces, it is not possible to flatten the surface without ...

Projections of the sphere

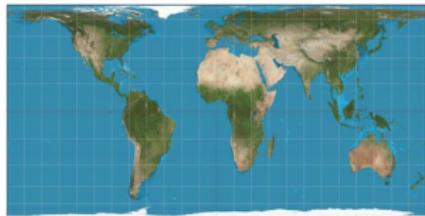
To represent the spherical earth on a flat sheet of paper, you have to make some compromises:



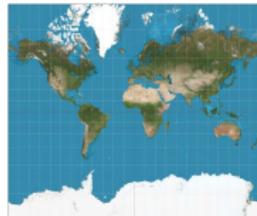
Goode-Homolsine Projection



Gnomonic Projection

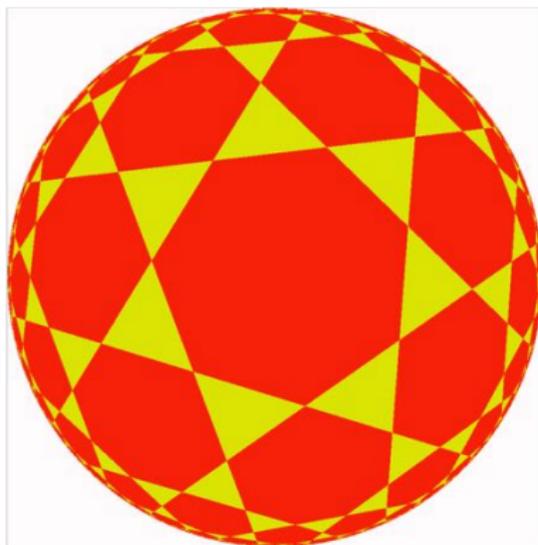


Hobo-Dyer Projection



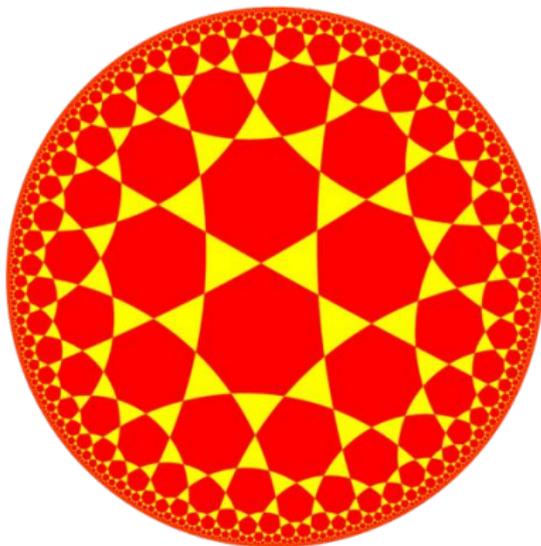
Mercator Projection

Representations of the hyperbolic plane



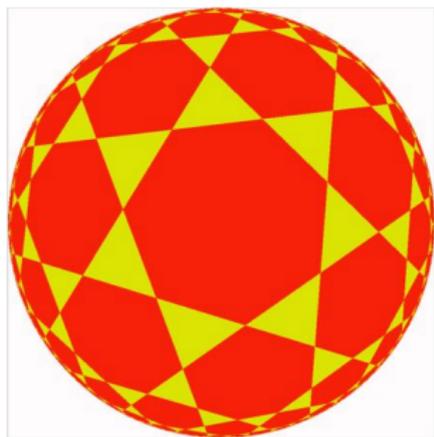
The Klein Model

Representations of the hyperbolic plane

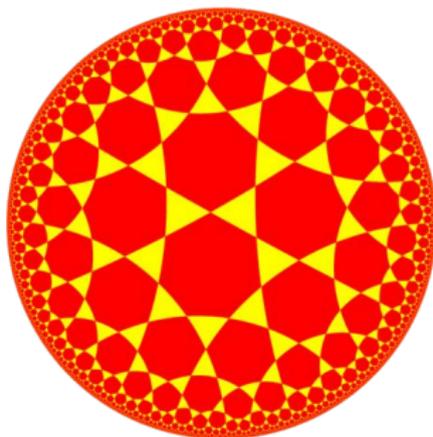


The Poincare Disk Model

Compare the Klein Model and the Poincare Model



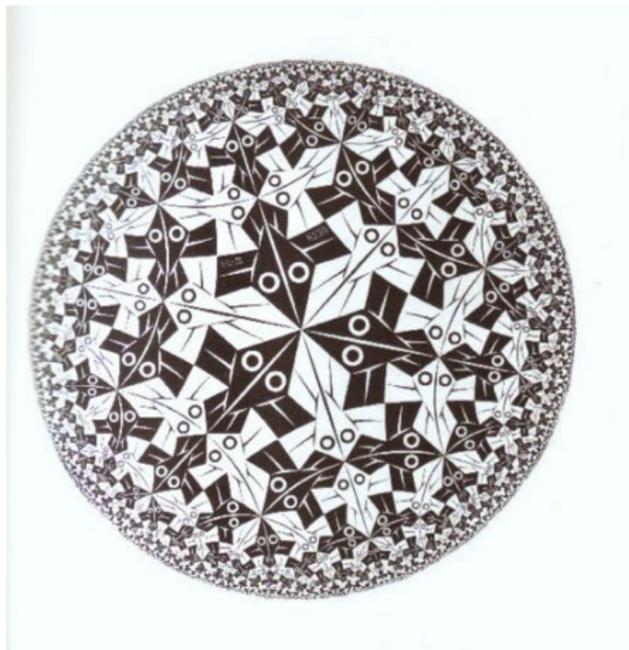
The Klein Model



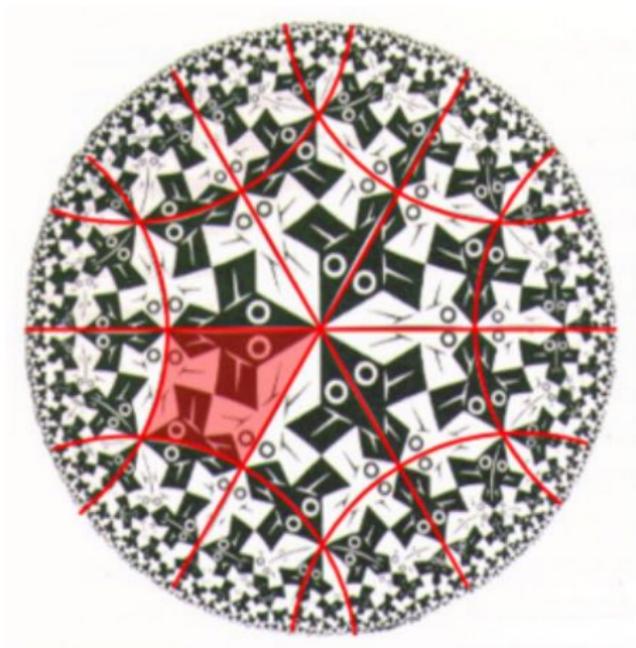
The Poincare Disk Model

What is the signature of this hyperbolic wallpaper pattern?

Escher's Circle Limit I



Escher's Circle Limit I



Escher's Circle Limit II



Escher's Circle Limit III

Circle Limit III, 1958

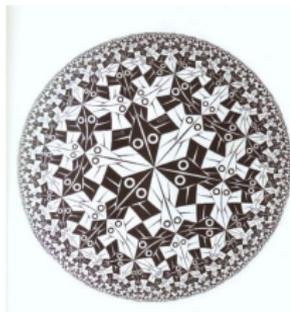
M.C. Escher, Cordon Art (c) 2002



Escher's Circle Limit IV



Escher's Circle Limits



Circle Limit III, 1958



M.C. Escher, Circle Art (c) 2002



Homework

1. Find the signatures of external link: Escher's four Circle Limit Patterns. Please specify if you are ignoring color or paying attention to color.
2. Use Jeff Week's external link: Kaleidotile software to make three patterns, including at least one hyperbolic pattern and at least one spherical pattern. Include the pictures and label each pattern with its signature. Note: to get patterns without mirror symmetry (e.g. 732 not *732) you can toggle View > Show Reflected Image. Note: in addition to using the available motifs, you can upload your own photos or any other pictures, by copying the picture and then clicking on the tiny yellow clipboard symbol.