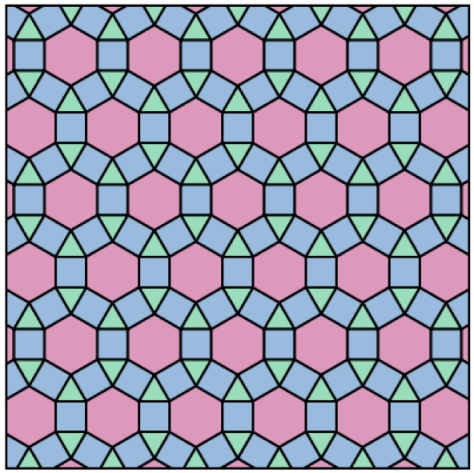
Part XXVIII

Tilings

This part is an introduction to tilings.

What is a tiling?

A collection of figures **tesellates**, or **tiles** the plane if it is possible to cover the entire surface of the plane with copies of the figures, with no gaps or overlaps.



Periodic and non-periodic tilings

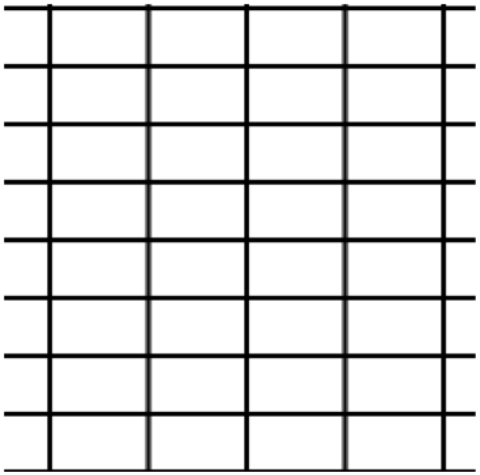
- A tiling is **periodic** if there is translation symmetry in two directions.
  - These tilings are the wallpaper patterns we have been studying.

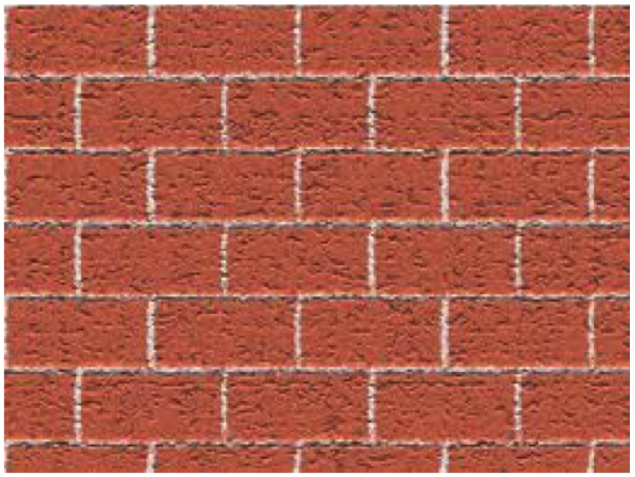
- A tiling is **non-periodic** or **aperiodic** if there is no translation symmetry.

Periodic tilings

- It is easy to create periodic tilings using triangles or rectangles.
- Draw some.

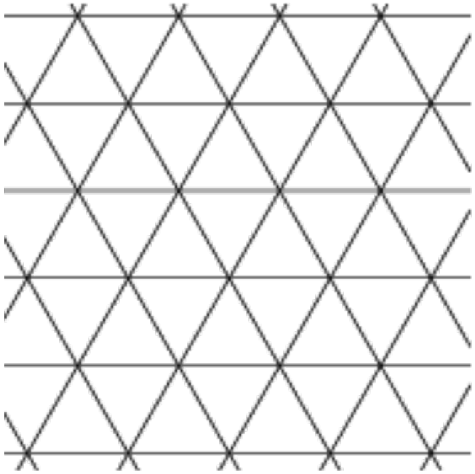
Periodic tilings

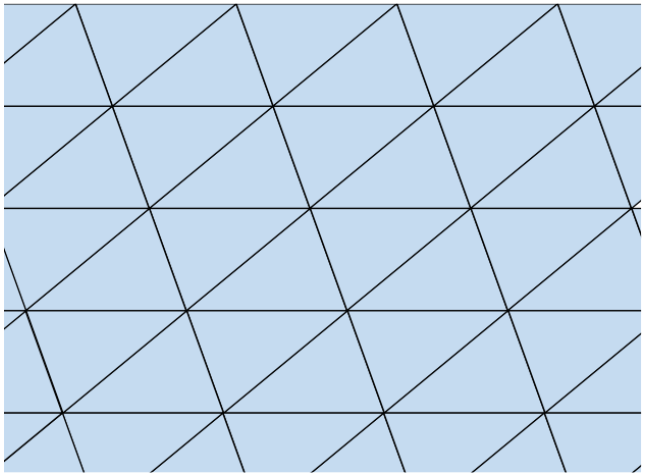


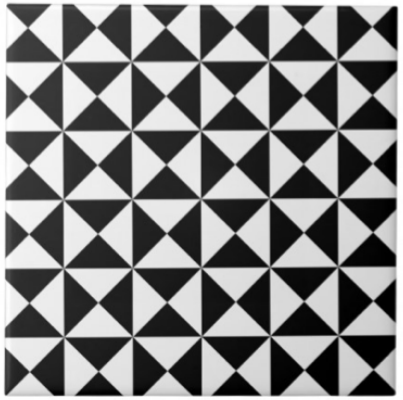


The first example is **edge-to-edge**. What about the second and third?

More periodic tilings







Which of these tilings are edge-to-edge?

Regular tilings

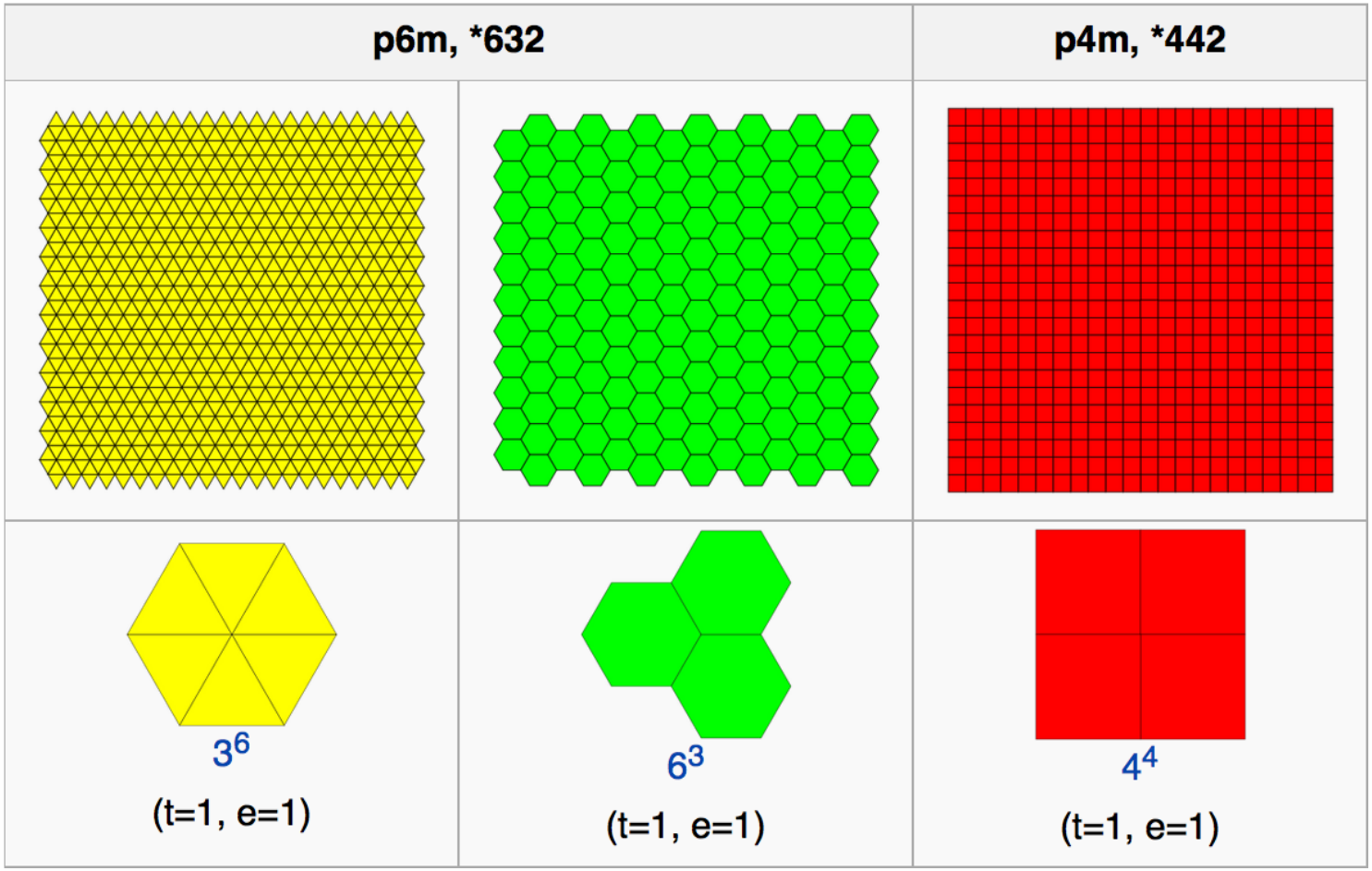
A tiling is **regular** or **Platonic** if

1. All tiles are regular polygons.
2. The tiling is edge to edge.
3. (All vertices look the same.)
4. There is only one type of tile.

What are some examples of regular tilings?

Regular tilings

There are only three types of regular tilings.



From Wikipedia.

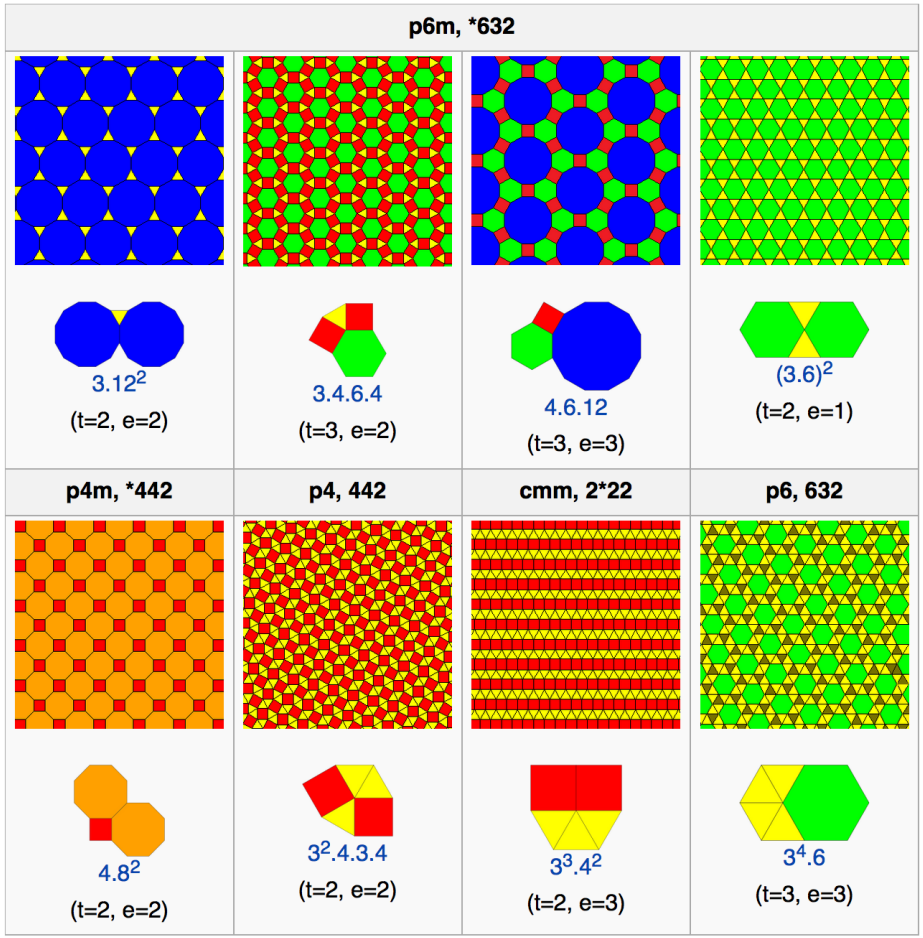
Semi-regular tilings

A tiling is **semi-regular** or **Archimedian** if

1. All tiles are regular polygons.
2. The tiling is edge to edge.
3. All vertices look the same.
4. There are two types of tile.

Semi-regular tilings

There are 8 types of semi-regular tilings.

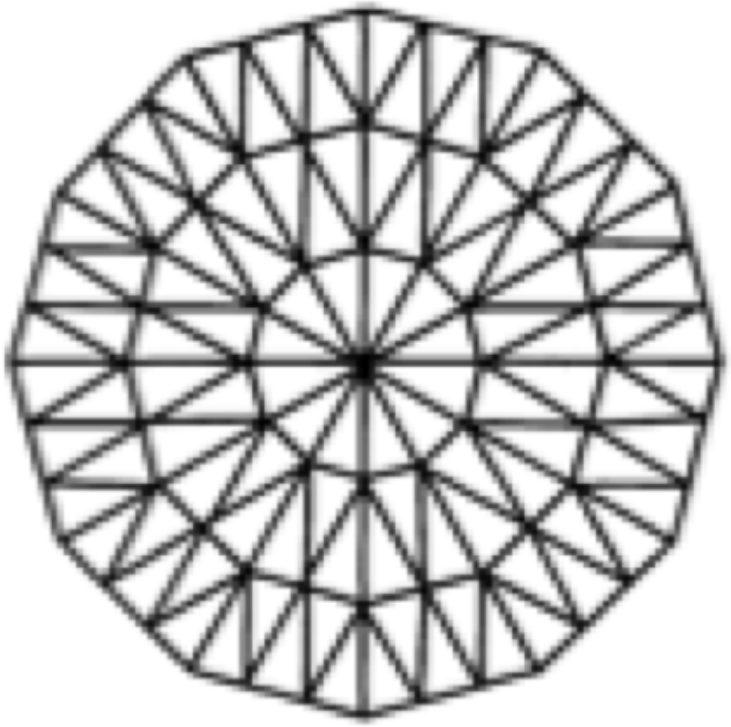


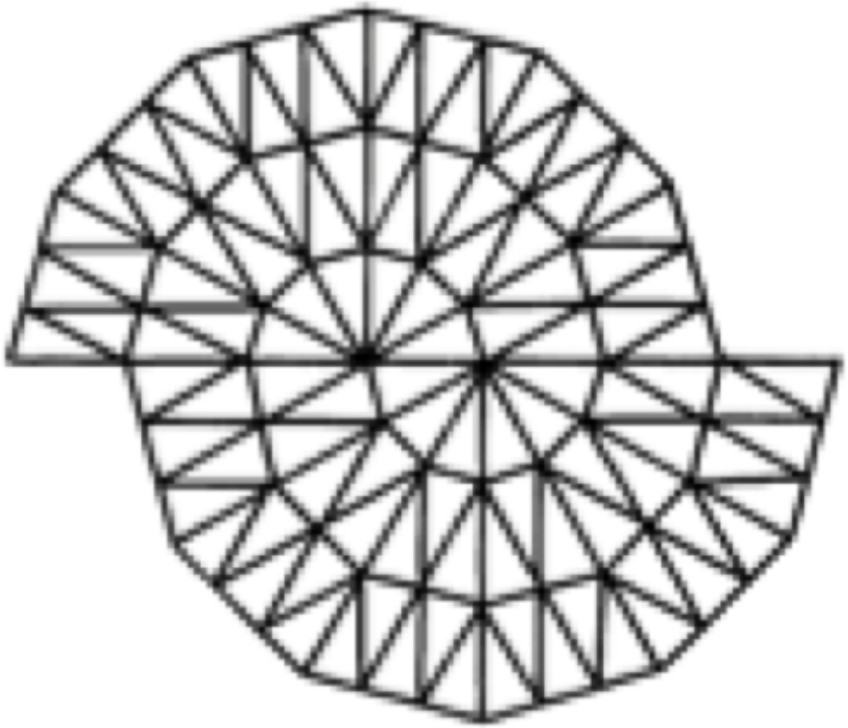
From Wikipedia.

Non-periodic tilings

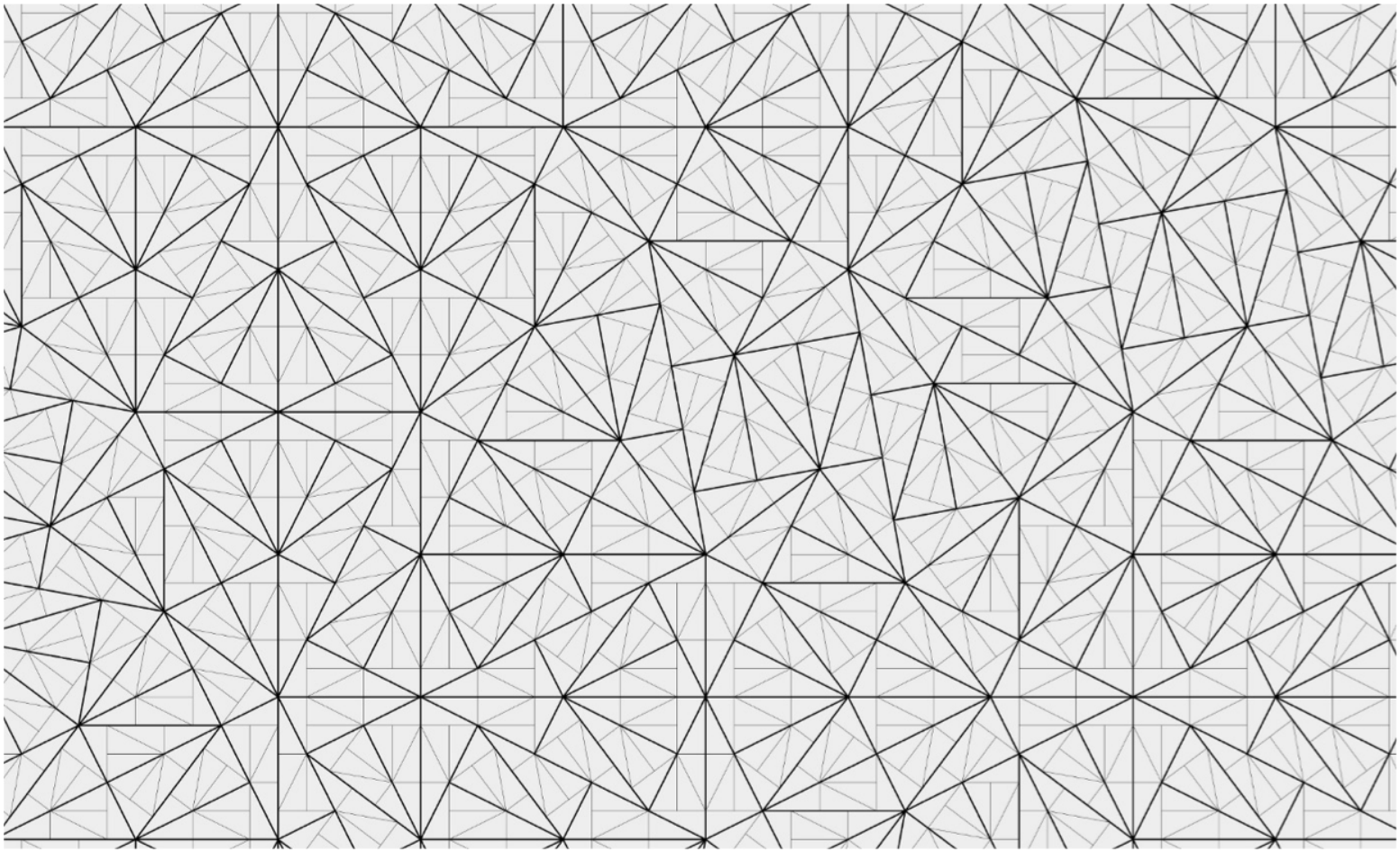
Is it possible to find an aperiodic tiling using rectangles or triangles?

Non-periodic tilings with triangles

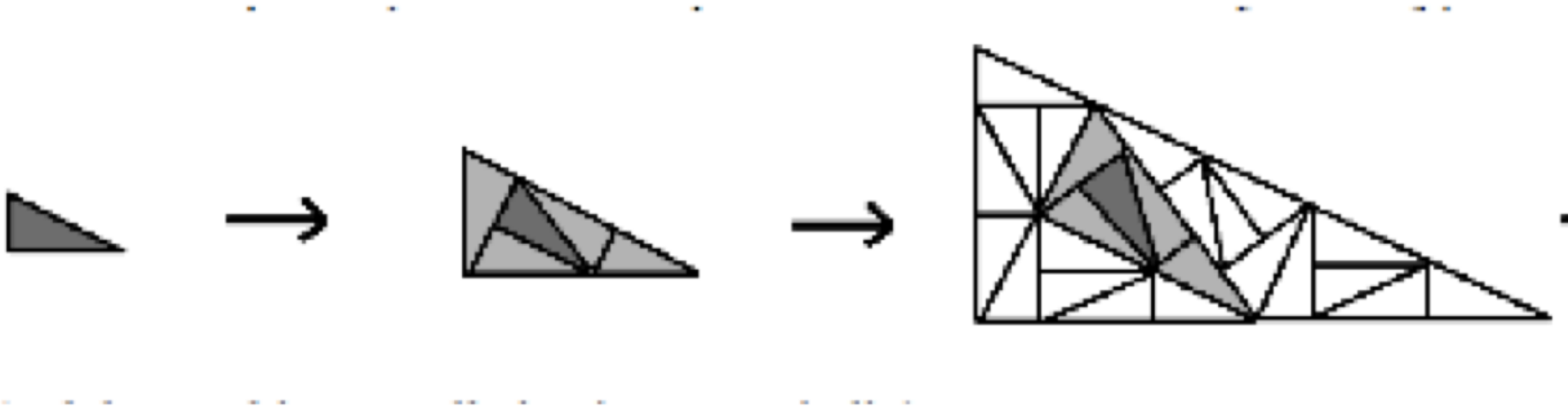




Conway's pinwheel tiling



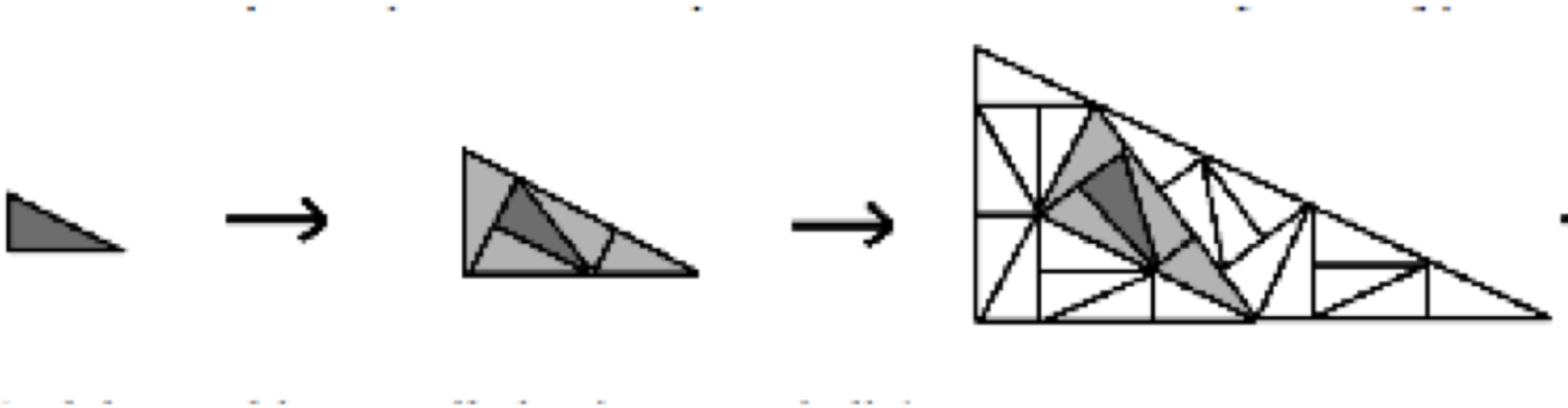
Conway's pinwheel tiling



John Conway

John Conway

Conway's pinwheel tiling



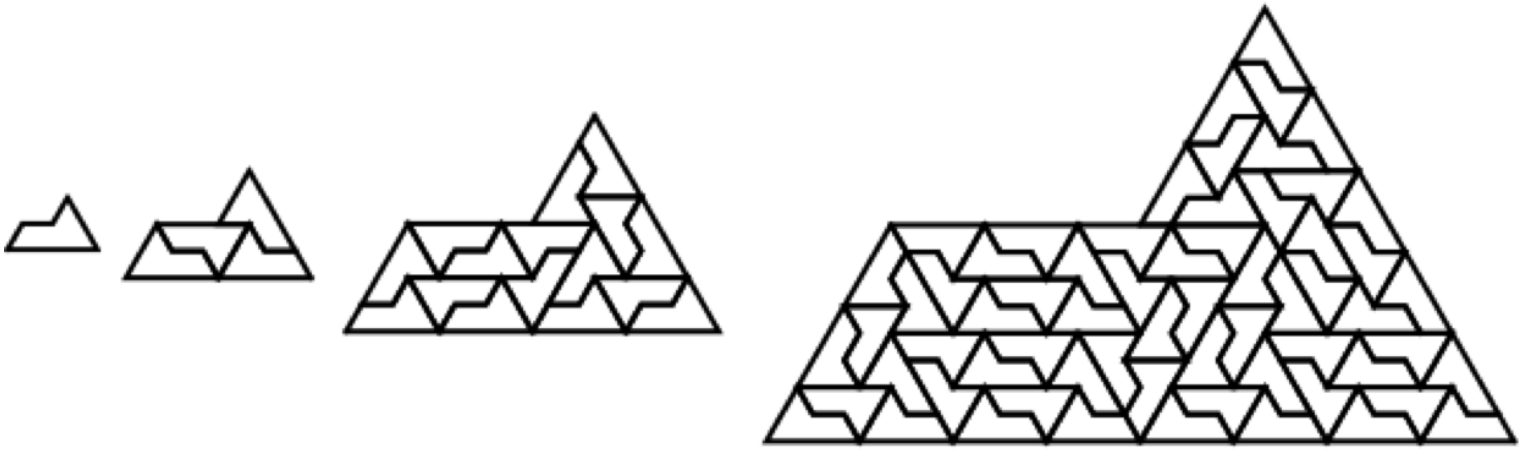
Self-replicating tilings

A tile self-replicates if a finite number of congruent copies of itself fit together to make a larger scaled copy of the tile. Sometimes these tiles are called "rep-tiles".

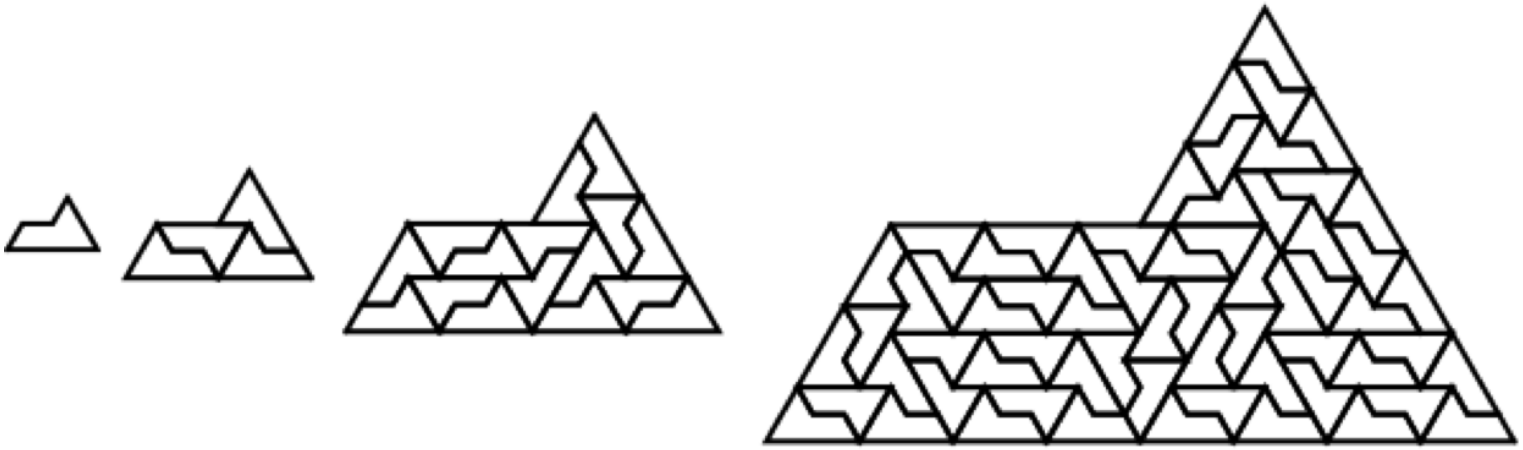
Self-replicating tiles, or rep-tiles, can be used to make interesting non-periodic tilings.

Experiment with rep-tiles

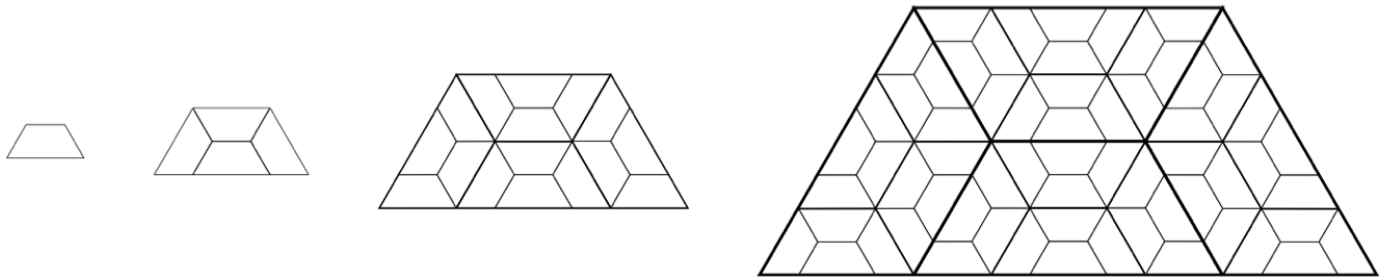
- Use the foam tiles or Geometers Sketchpad to make aperiodic tilings, using the substitution method.
- Can you also make periodic tilings with these tiles?



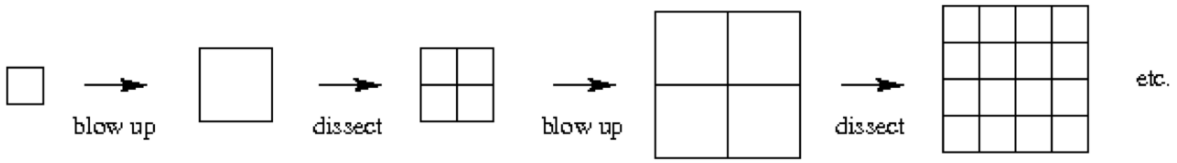
IDeflating the sphinx



Deflating the trapeziod

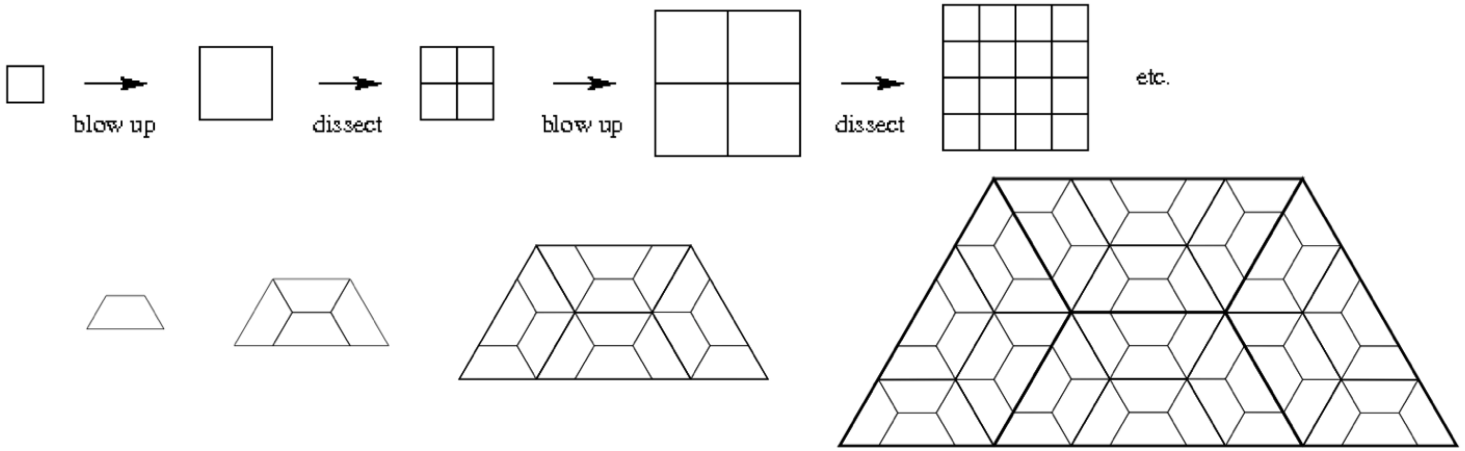


Deflating the square

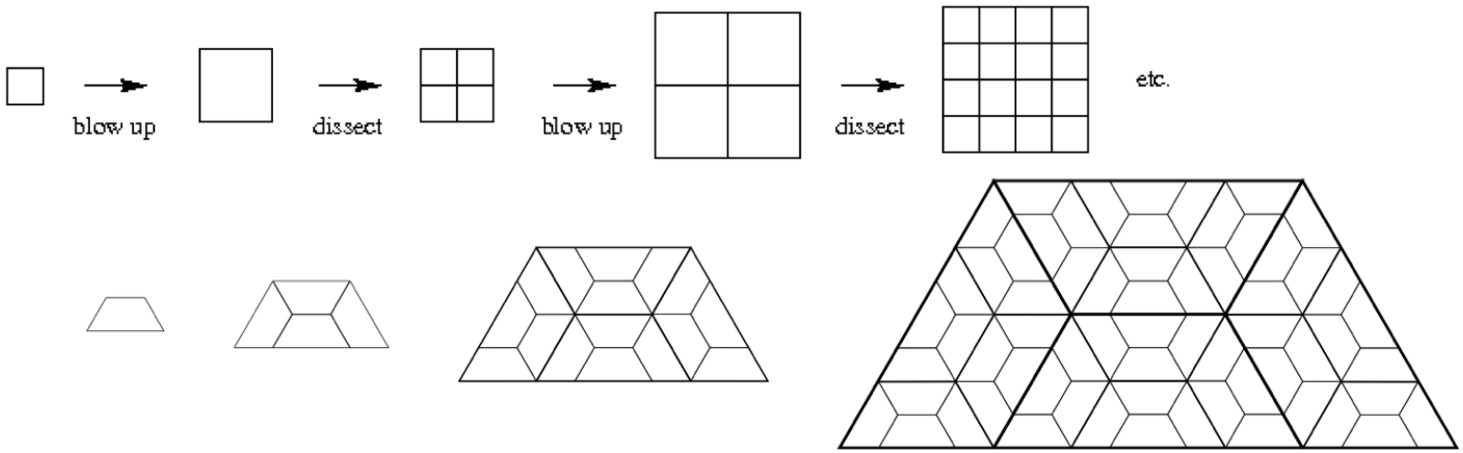


Periodic vs. non-periodic self-similar tilings

- The square rep-tiling is periodic
- The Sphynx, trapezoid, and L rep-tilings are aperiodic
- What makes them different?



What makes them different?



- The square tiling can be composed (joined) into larger squares made of 4 little squares in different ways.
- The trapezoid tiling can be composed (joined) into larger trapezoids made of 4 little trapezoids in a unique way.

Unique composition makes the tiling aperiodic

- Suppose there is a translation symmetry
- It will also have to be a translation symmetry of the larger composed tiles
- Keep composing ... the translation symmetry will still be a symmetry of larger and larger tiles
- Eventually the size of the tiles will be bigger than the translation length itself
- It is impossible to have a translation symmetry of a tiling whose length is smaller than the size of the tiles.
- Contradiction!!