

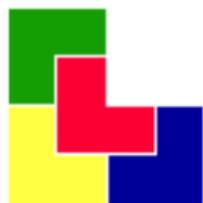
Part XXX

Penrose Tiles

This is an introduction to Penrose Tiles.

Aperiodic tilings

Last time, we looked at tiles that could tile the plane in an aperiodic way (no translation symmetry).



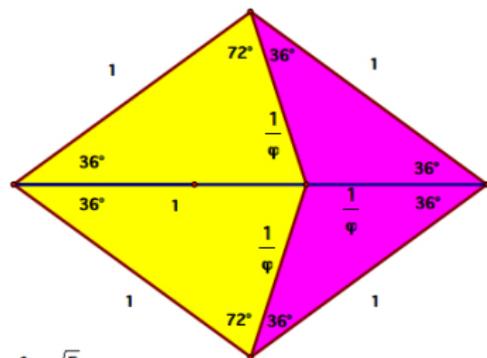
All of these tiles can also tile the plane in a periodic way.

Aperiodic tiles

Is it possible to find tiles that tile the plane only in non-periodic ways?

Penrose tiles

These are the “kite” and “dart” Penrose tiles.

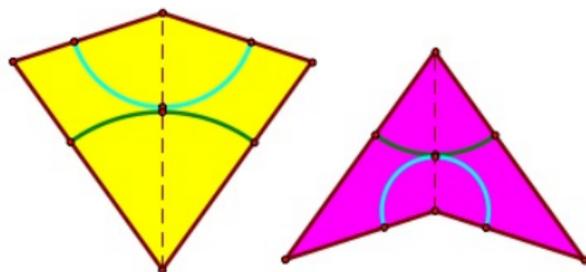


$$\varphi = \frac{1 + \sqrt{5}}{2}$$

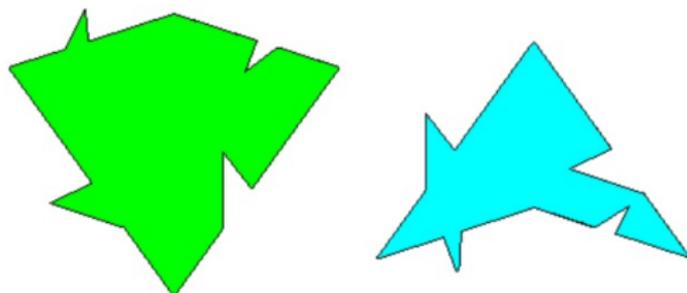
What do you notice about their angles and side lengths?

Penrose tiles, with side-matching condition, are aperiodic tiles

Condition: edges must be placed so that the blue circle match up and the green circles match up. This could be accomplished instead by notching the tiles.

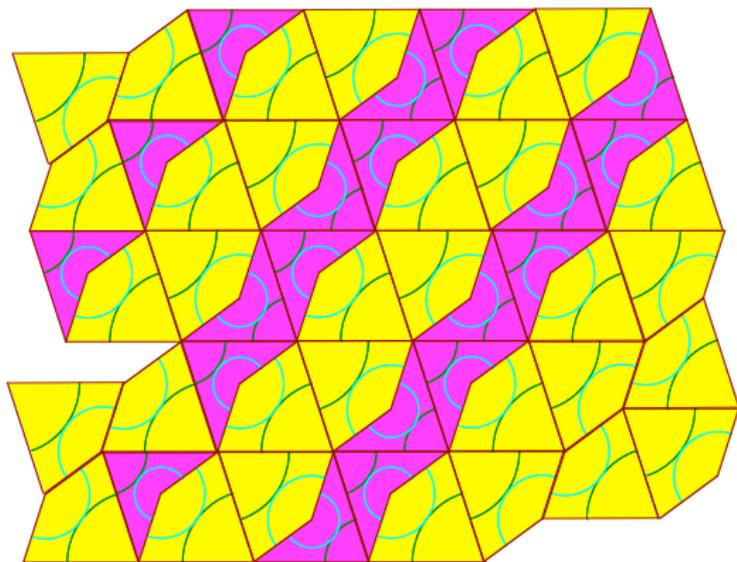


This could be accomplished instead by notching the tiles.



Build a Penrose tiling

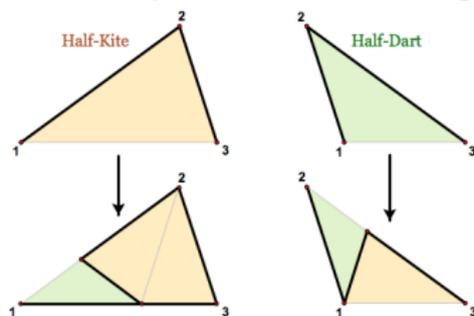
Use the template to cut out Penrose tiles and build the biggest tiling that you can.



Count the number of kites and the number of darts in your tiling.

Self-replication for Penrose Tiles

- ▶ The Penrose Tiles have their own self-similarity.
- ▶ Use the Geometers Sketchpad Penrose Tiling Sketch to inflate and decompose Penrose tilings



- ▶ Start with any small configuration of kites and darts
- ▶ Do at least two levels of inflation
- ▶ Count the number of kites and darts at the end.

Why are Penrose Tiles Aperiodic?

Self-replication

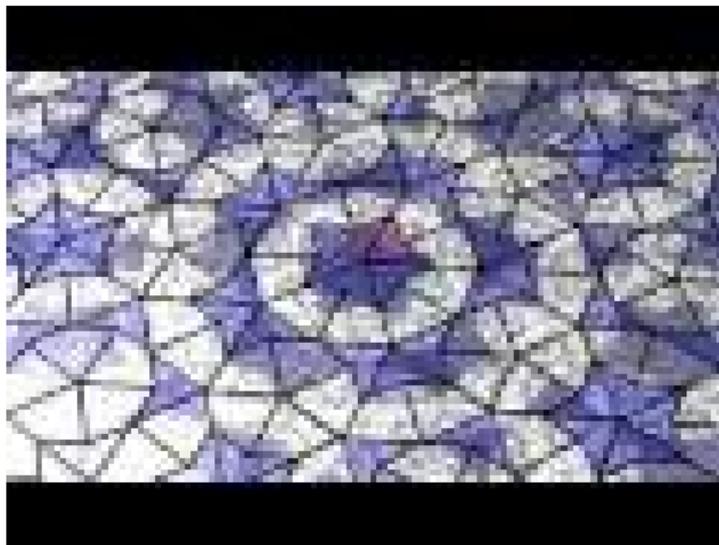
- ▶ As with the other reptiles, Penrose tiles have the property of self-replication.
- ▶ As with the other reptiles, Penrose tiles have unique composition – they can only be joined into larger, self-similar tiles in a unique way.
- ▶ These properties give one way of proving that they tile in an aperiodic way.

Why are Penrose Tiles Aperiodic?

The Golden Ratio

- ▶ Fact: as the tiled region gets large, the ratio of kites to darts approaches the Golden Ratio $\phi = \frac{1 + \sqrt{5}}{2} = 1.6180339887\dots$
- ▶ Why is this true for tilings made by deflation?
- ▶ Could the ratio of kits to darts approach this number if the tiling were periodic?

Animated Penrose Tile Movie



From Maurizio Paolini and Alessandro Musesti, the Department of Mathematics and Physics, Catholic University, Brescia (Italy)