Part III

Reflections of Reflections

The goal of this section is to analyze isometries in terms of reflections.

Reflections of reflections

- Suppose we have two parallel mirror lines, and we do a reflection through the first mirror line followed by a reflection through the second mirror line. What is the isometry that results?

- Suppose we have two mirror lines that are NOT parallel (they intersect), and we do a reflection through the first mirror line followed by a reflection through the second mirror line. What is the isometry that results?

Mirrors are available, or you may want to use Sketchpad to help.

Angles and distances for products of two reflections

Reflect the point A through the mirror BC and draw the image point A'. Then reflect A' through mirror DE and draw its image A''. What is the distance between A and A'' in terms of the distances d and x?
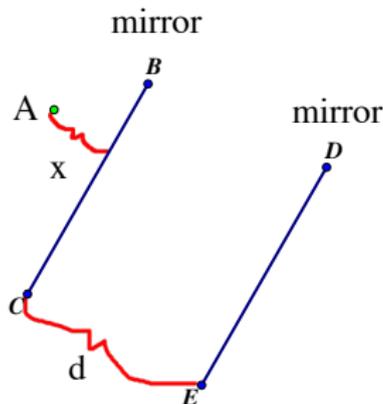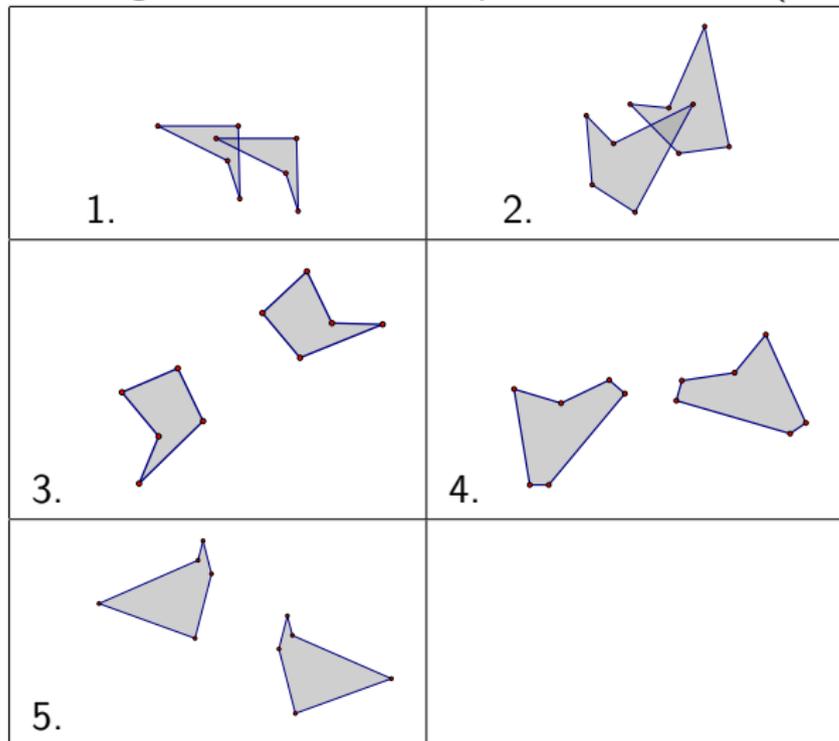

Reflect the point A through the mirror CF and draw the image point A'. Then reflect A' through mirror DE and draw its image A''. What is the angle $\angle ABA''$ in terms of the angles δ and θ?
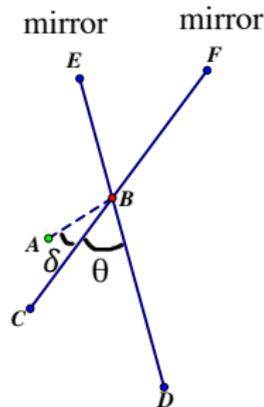
What happens if you do the reflections in the other order?

Can you get from here to there with reflections?

Each figure below shows a pair of isometric (congruent) polygons.

For each figure, find a sequence of no more than *three* reflections so that one polygon ends up coinciding with the other polygon.

Con you always get from here to there with at most three reflections?

- ▶ What kinds of isometries result from a product (i.e. a sequence) of 2 reflections? From a product of 3 reflections?

- ▶ Can *any* isometry can be viewed as a product of 1, 2, or 3 reflections?
    - ▶ If so, find an *algorithm* to locate the mirrors (i.e. give instructions that could be applied to any pair of isometric figures).
    - ▶ If not, draw a pair of isometric figures for which there is no such sequence.

Homework

1. In the slide on parallel and intersecting mirrors, what happens if you do the reflections in the other order? Do you get the same result or something different? Explain.

2. Suppose you have 3 parallel mirrors. Suppose the first two mirrors are distance d apart and the next two are distance e apart. What isometry results from reflecting through the each of the three mirrors in order? Be precise: for example, if the answer is a translation, give the direction and distance of translation, and if the answer is a reflection, give the position of the mirror line, etc. Your answer will be in terms of d and e and the positions of the original mirrors.