Expected Value

After completing this section, students should be able to:

- Compute expected value in simple games of chance and other simple examples.

Expected Value Intro Video

Example. Suppose I offer you the following game: You roll of a fair 6-sided die.

- If a 6 comes up, you win $6.00

- If a 5 comes up, you win $3.00

- If a 4 comes up, you don't win or lose anything.

- If a 1, 2, or 3 comes up, lose $4.00

Do you want to play?

Example. An American roulette game has 38 numbers, 1 - 36 plus 0 and 00. Some numbers are red and some are black, but 0 and 00 are neither of these colors. If you bet a dollar on red and win, you get your dollar back plus another dollar. But if you lose, you lose your dollar. What is your expected win or loss for this game?

Example. An American roulette game has 38 numbers, 1 - 36 plus 0 and 00. A single number pays 35 to 1, which means that if you bet a dollar on a single number and win, you get your dollar back plus another $35. If you lose, you lose your dollar. What is your expected win or loss for betting on a single number?

END OF VIDEO

Expected Value Examples

Example. A card game goes like this: You pay $1 to play, and you draw one card from a 52-card deck. Such a deck has four suits (hearts, diamonds, clubs and spades), and there is one of each of the following cards for each suit: 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king and ace. When you draw an ace, you win $4. If you draw any face card (jack, queen, or king) you win $2. Otherwise, you win nothing.

If you play this game a large number of times, how much do you expect to gain (or lose) on average per game, taking into account the dollar to play? Choose the closest answer.

A. Lose $1

B. Lose 75 cents

C. Lose 50 cents

D. Lose 25 cents

E. Come out even

F. Win 25 cents

G. Win 50 cents

H. Win 75 cents

I. Win $1

Example. The car insurance industry estimates that the probability of your having an automobile collision this year is about 6%. Repair costs vary dramatically, but for simplicity, let's assume that it costs an average of $4000 to repair a car that has been in a collision. You pay $300 this year for insurance.

(a) What is the expected annual profit of the insurance policy for the company per customer?

(b) What is the expected cost for an average customer if they buy the insurance policy?

(c) What is the expected cost for an average customer if they don't?

Example. Suppose that on a standardized test of multiple choice problems, each problem has 5 answer choices. If you get the answer right, you get 1 point, if you leave it blank, you get 0 points, but if you get it wrong, you lose 1/4 point. This scoring system is supposed to discourage wild guessing. Does it?

What is the expected value for the points you receive on a problem if you guess at random after eliminating one answer that you know is incorrect?

Example. Suppose a company charges an annual premium of $100 for an insurance policy for minor injuries. Actuarial studies show that in case of an injury claim, the company will pay out an average of $900 for outpatient care and an average of $3000 for an overnight stay in the hospital. They also determine that, on average, each year there are five claims made that result in outpatient care for every 1000 policies and three claims made that result in an overnight stay out of every 1000 policies. Assume for super simplicity that there are no deductibles or out of pocket costs if you are on the insurance policy, and that the company has no overhead costs.

Can the insurance company expect business to be profitable?

Is it in the customer's interest to buy this insurance, considering finances only?

Example. Say if you cheat, you will get an 85 on the test instead of a 65. But say there is a 25% chance of getting caught, and if you are caught you will get a zero on the assignment. Under these assumptions, from a cold mindset devoid of ethical considerations, is it to your advantage to cheat?

Do you think these assumptions are reasonable?

Extra Example. Use a spreadsheet to simulate the game on the before class video. Simulate a large number of games, like 200, and find the average gain / loss per game.

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Extra Example. Use a spreadsheet to simulate the card game example at the beginning of class. Simulate a large number of games, like 500, and find the average gain / loss per game.

A card game goes like this: You pay $1 to play, and you draw one card from a 52-card deck. Such a deck has four suits (hearts, diamonds, clubs and spades), and there is one of each of the following cards for each suit: 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king and ace. When you draw an ace, you win $4. If you draw any face card (jack, queen, or king) you win $2. Otherwise, you win nothing.