

Parabolas

After completing this section, students should be able to:

- Define a parabola as the set of points that are equidistant from a point (its focus) and a line (its directrix).
- Given the focus and directrix of a parabola, or the focus and vertex, or the vertex and directrix, write down its equation in the form $(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$.
- Graph a parabola given in the form $(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$ and locate its focus, directrix, and axis of symmetry.
- Given an equation of a parabola in a general form like $4x^2 - 20x - 8y + 57 = 0$, rewrite it in a standard form $(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$.

Example. Find the equation of the points equidistant from the point $(0, p)$ and the horizontal line $y = -p$.

Example. Find the equation of the points equidistant from the point $(p, 0)$ and the vertical line $x = -p$.

Example. Find the equation of a parabola with vertex at (h, k) , assuming the parabola opens up or down.

Example. Find the equation of a parabola with vertex at (h, k) , assuming the parabola left or right.

Example. Find the equation of a parabola with vertex at $(2, 4)$ and focus at $(-1, 4)$. Graph the parabola.

Review. Which of the following represents the equation of a parabola?

A. $y = ax^2 + bx + c$

B. $y = a(x - h)^2 + k$

C. $4p(y - k) = (x - h)^2$

D. $(x - h)^2 + (y - k)^2 = r^2$

Review. Which of the following represents the equation of a parabola that opens to the left?

A. $8(y - 5) = (x + 2)^2$

B. $y + 1 = -4(x - 3)^2$

C. $12(x + 6) = (y - 5)^2$

D. $x - 3 = -\frac{1}{2}(y + 2)^2$

Fill out the chart based on the figures.

	Opens up or down	Opens left or right
Equation		
Vertex		
Focus		
Directrix		
Axis of symmetry		
Graph ($p > 0$)		
Graph ($p < 0$)		

Example. Find the equation of a parabola with vertex at $(1, 6)$ and directrix at $x = 9$.

Extra Example. Find the equation of a parabola with focus at $(-3, 2)$ and directrix at $y = 8$.

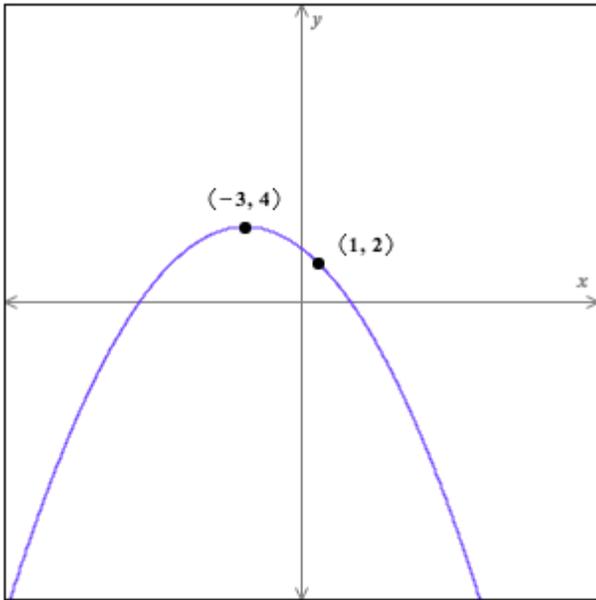
Example. Find the focus, directrix, and vertex of this parabola. Then graph the parabola.

$$x^2 - 6x - 2y + 1 = 0$$

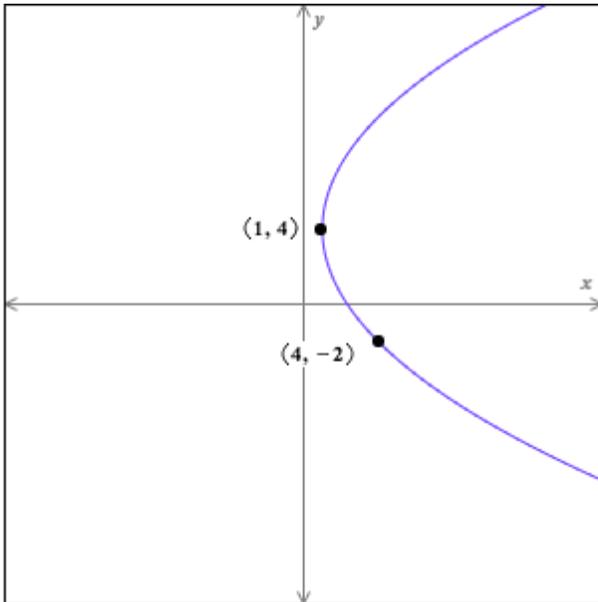
Extra Example. Find the focus, directrix, and vertex of this parabola. Then graph the parabola.

$$-y^2 + 8y + 5x - 31 = 0$$

Example. Find the equation of the parabola.



Extra Example. Find the equation of the parabola.



Example. A satellite dish has the shape of a paraboloid that is 10 feet across at the opening and is 3 feet deep. At what distance from the center of the dish should the receiver be placed to receive the greatest intensity of sound waves?

Example. The Golden Gate Bridge, a suspension bridge, spans the entrance to San Francisco Bay. Its towers rise 526 feet above the road and are 4200 feet apart. The bridge is suspended from two cables. The cables are parabolic in shape and touch the road surface at the center of the bridge. Using the location where the cables touch the road surface as $(0,0)$, find the height of the cable 200 feet from the center of the bridge.

Extra Example. Which of the following equations are equivalent to $y = \frac{3}{2}x^2 + 15x + \frac{69}{2}$?

A. $y = \frac{3}{2}(x + 5)^2 - 3$

B. $(x + 5)^2 = \frac{2}{3}(y + 3)$

C. $3x^2 + 30x - 2y + 69 = 0$

D. None of these.

Ellipses

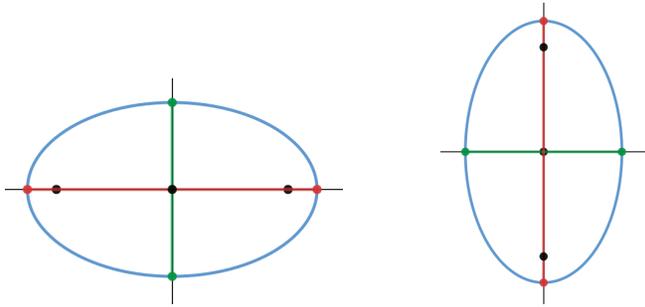
After completing this section, students should be able to:

- Define an ellipse in terms of distances to the two foci.
- Given an equation of an ellipse in a form like $\frac{(x + 4)^2}{16} + \frac{(y - 1)^2}{4} = 1$, find its center, vertices, foci, and major and minor axes.
- Given an equation in a form like $4x^2 + 9y^2 + 8x - 36y = 6$, complete the square to put it in the form $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$
- Given an equation in the form the form $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$, draw its graph.
- Write down the equation for an ellipse from its graph.
- Find the equation of an ellipse from the coordinates of its foci and one of its vertices, or similar information.
- Solve applications problems involving ellipses.

Recall: A circle can be defined as the set of points ...

Note. An ellipse can be defined as the set of points ...

Features of an ellipse



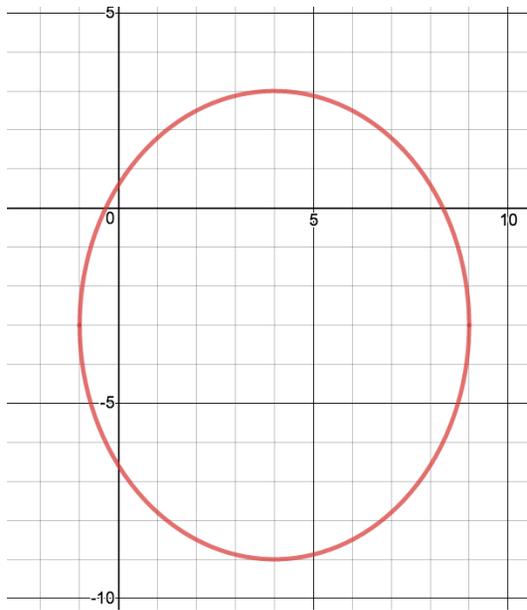
Example. Find the equation of an ellipse with foci at $(-c, 0)$ and $(c, 0)$ and vertices at $(-a, 0)$ and $(a, 0)$.

Summary: For $a > b$, the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents ...

For $a > b$, the equation $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ represents ...

What about an ellipse centered at (h, k) ?

Example. Write the equation of the ellipse drawn. Find its foci.



Review. Which of the following represents the equation for an ellipse?

A. $9x^2 + 4y^2 = 36$

B. $\frac{x^2}{25} - \frac{y^2}{100} = 1$

C. $4x^2 + 6y^2 + 8x + 12y = 15$

D. $\frac{(x - 3)^2}{4} + \frac{(y + 1)^2}{9} = 1$

Example. Graph the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Example. Graph the ellipse $\frac{(x - 3)^2}{4} + \frac{(y + 1)^2}{9} = 1$.

What are the center, vertices, and foci?

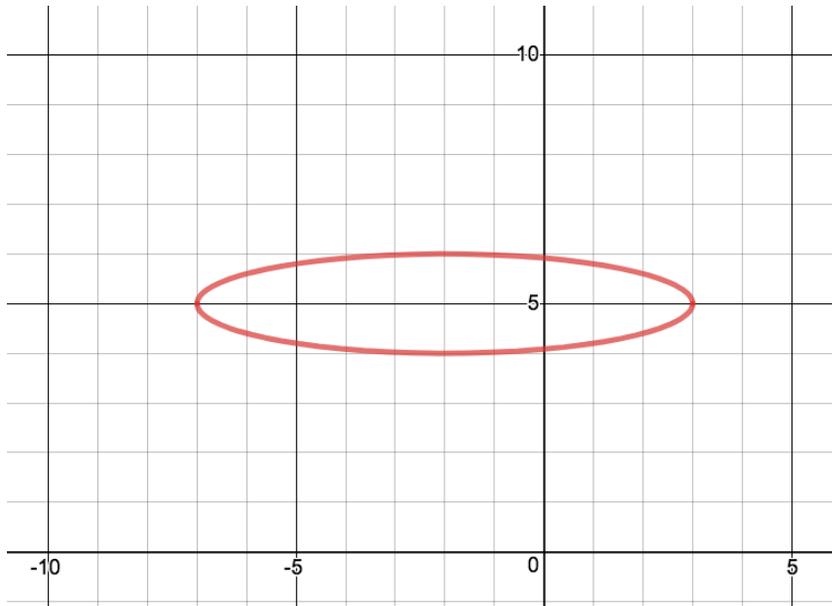
Example. Graph the ellipse $4x^2 + 9y^2 - 16x + 18y - 11 = 0$.

What are the center, vertices, and foci?

Example. Find an equation for the ellipse with foci at $(-4, 2)$ and $(-4, 8)$ and vertex at $(-4, 10)$.

Extra Example. Find the equation of an ellipse with a minor axis of length 6 and a center at $(-3, 3)$ and a vertex at $(1, 3)$

Extra Example. Write the equation of the ellipse drawn.



Extra Example. A one-way road passes under an overpass in the form of half of an ellipse, 15 feet high at the center and 20 feet wide. Assuming a truck is 12 feet wide, what is the height of the tallest truck that can pass under the overpass?

Extra Example. A satellite is placed in elliptical earth orbit with a minimum distance of 160 km and a maximum distance of 16000 km above the earth. Find the equation of the orbit of the satellite, assuming that the center of the earth is at one focal point and that the radius of the earth is 6380 km.

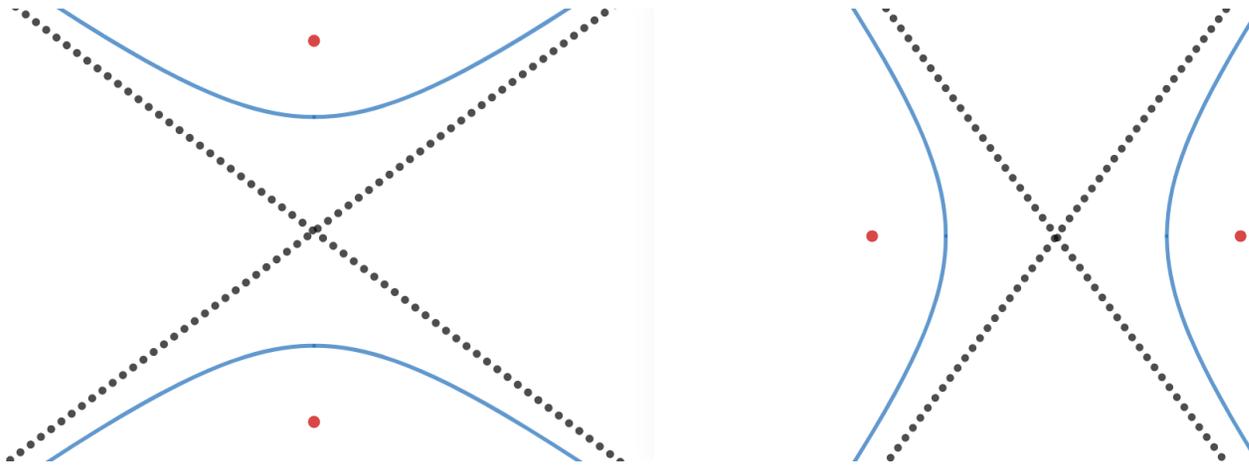
Hyperbolas

After completing this section, students should be able to:

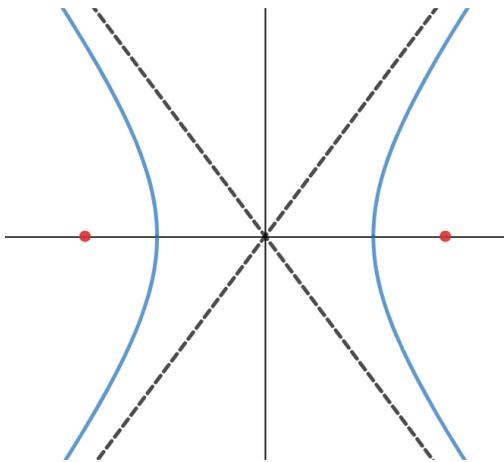
- Define a hyperbola in terms of the distances to the two foci.
- Given an equation of a hyperbola in a form like $\frac{(x + 4)^2}{16} - \frac{(y - 1)^2}{4} = 1$, find its vertices and center.
- Given an equation in a form like $4x^2 - 9y^2 + 8x - 36y = 6$, complete the square to put it in the form $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$
- Given an equation in the form the form $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$, draw its graph.
- Write down the equation for a hyperbola from its graph.
- Find the equation of a hyperbola from the coordinates of its foci and one of its vertices, or similar information.
- Solve applications problems involving hyperbolas.

Definition. A hyperbola is the set of points (x, y) such that ...

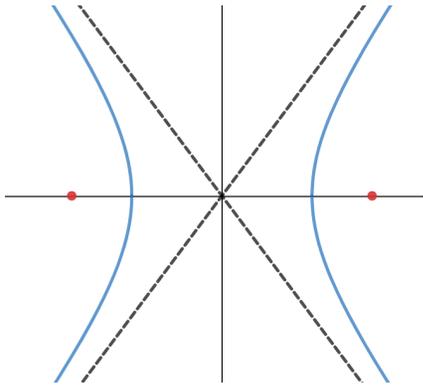
Features of a hyperbola



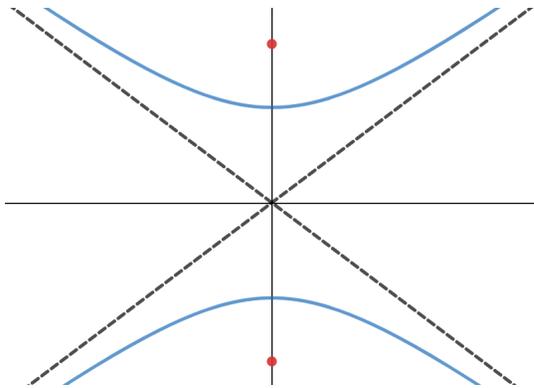
Example. Find the equation of an hyperbola with foci at $(-c, 0)$ and $(c, 0)$ and vertices at $(-a, 0)$ and $(a, 0)$.



Summary: The equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ represents ...



The equation $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ represents ...



What about an hyperbola centered at (h, k) ?

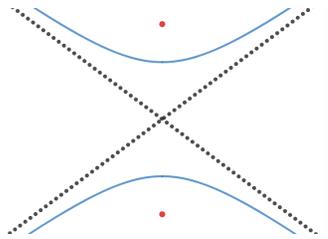
Example. Graph the hyperbola $\frac{(x - 6)^2}{4} - \frac{(y + 3)^2}{25} = 1$.

Find its center, vertices, foci, and asymptotes.

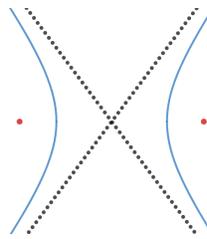
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Review. For the equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

1 Which way is the hyperbola oriented?



OR



2 The center is at ...

3 a represents ...

4 c represents ...

5 c is related to a and b by the equation ...

6 The slope of the asymptotes is:

7 For hyperbolas, a is (circle one) the larger number / the number that goes with the positive term. (What about for ellipses?)

Which answers change for the equation $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$?

Example. Find the center, vertices, foci, and asymptotes for $\frac{(x - 1)^2}{9} - \frac{(y + 3)^2}{4} = 1$.
Graph the hyperbola.

Example. Find the center, vertices, foci, and asymptotes for the hyperbola $y^2 - 6y - 4x^2 - 32x - 59 = 0$. Graph the hyperbola.

Example. Write the equation of a hyperbola with foci at $(0, -3)$ and $(-4, -3)$ and a vertex at $(-3, -3)$.

Extra Example. Find an equation of the hyperbola having foci at $(3, 8)$ and $(3, 12)$ and vertices at $(3, 9)$ and $(11, 3)$

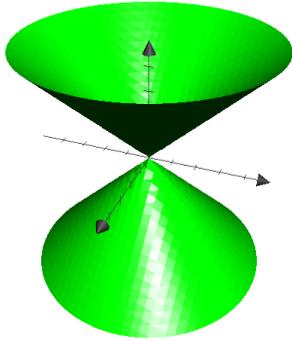
Extra Example. Shearon Harris Nuclear Plant in North Carolina has a hyperboloid cooling tower 526 feet tall. Assuming that the Harris cooling tower is 400 feet wide at its base and is 250 feet wide at its narrowest point 400 off the ground. What is the equation of the hyperbola that models the shape of this tower?



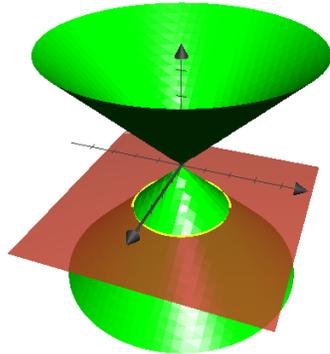
Extra Example. What is the equation of the hyperbola that models the light pattern on the wall?



Question. What shapes can you get by slicing a double cone with a plane?

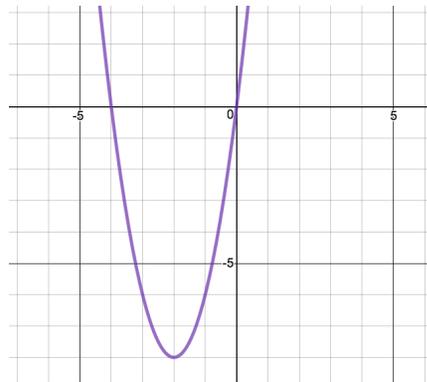
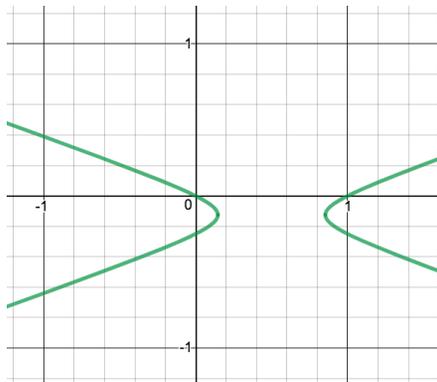
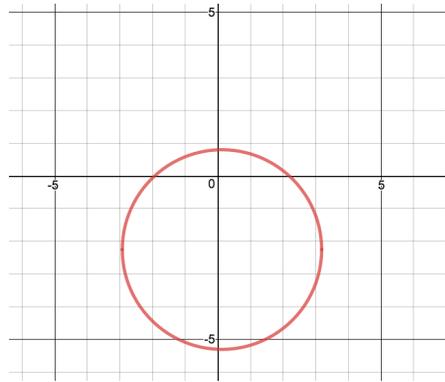
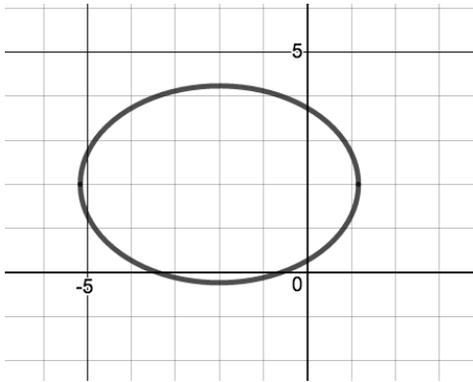


For example, if you cut it with a horizontal plane, you will get a circle.



If you tilt the plane, or make it vertical, what other shapes can you get?

Example. Identify each conic section.



Example. Identify each conic section.

A. $x^2 - 8y^2 - x - 2y = 0$

B. $2x^2 - y + 8x = 0$

C. $x^2 + 2y^2 + 4x - 8y + 2 = 0$

D. $4x^2 + 4y^2 - x + 18y - 17 = 0$