§8 Symmetry and isometries

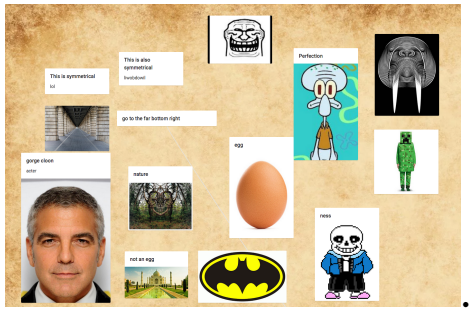
The goal of this section is to define symmetry and isometries.

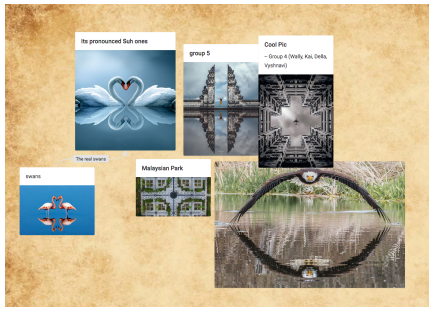
Supplies:

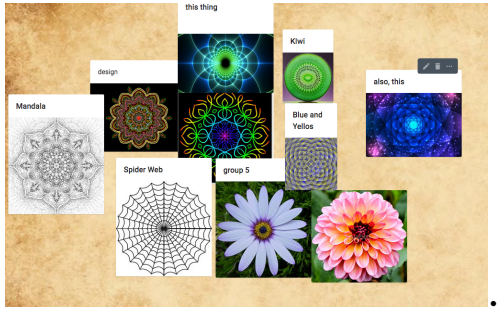
- large cut outs of cartoon characters that are flipable

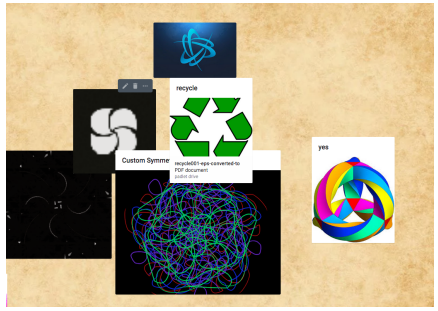
Find an image that shows symmetry and post it on Padlet.

1. Group the patterns based on which ones have the same or related types of symmetry.
2. Describe the types of symmetry you see.
3. How would you define "symmetry"?

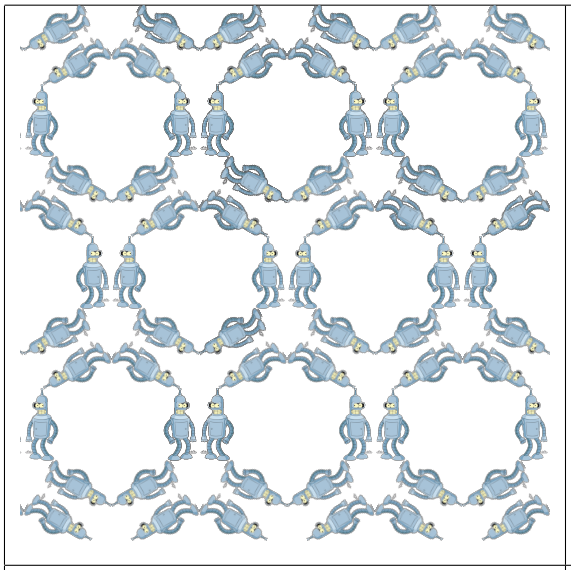
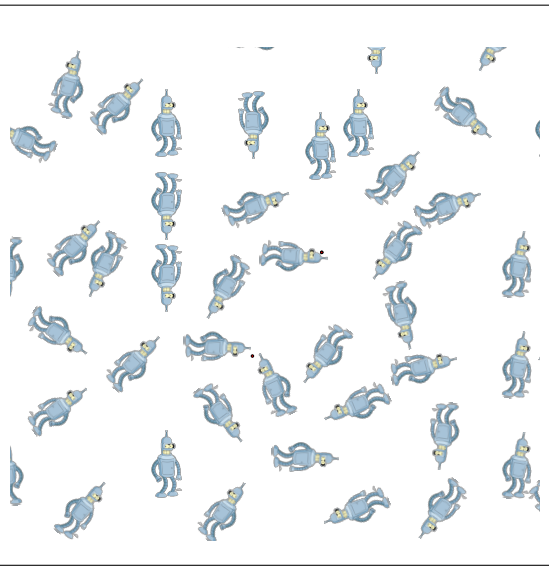
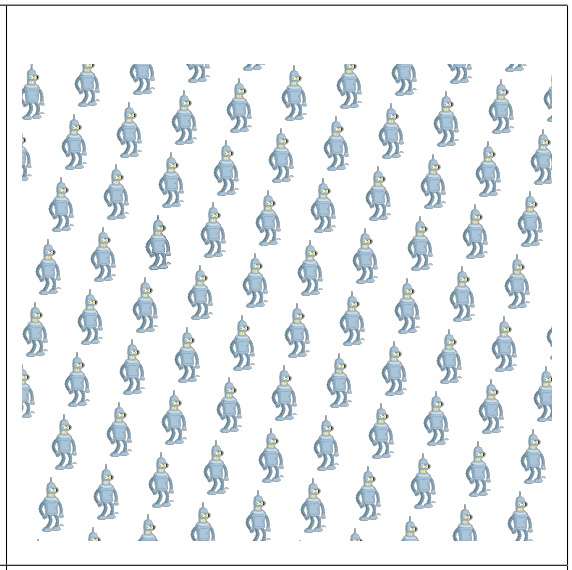








Which figure would you say has the most symmetry? the least symmetry? Why?

		
A	B	C

Isometries

- An *isometry* of the plane is:

- Isometries of the plane are also called:

- Name and describe as many isometries of the plane as you can think of.

Anatomy of Isometries

A *fixed point* of an isometry is a point that ...

Which isometries have fixed points and which do not?

Guess the definitions of the following terms:

- rotocenter
- mirror line
- translation vector
- glide line

An isometry is orientation reversing if ...

An isometry is orientation preserving if ...

Which isometries are orientation preserving and which are orientation reversing?

- If a point of the object starts out a distance d from the rotocenter of a rotation, then how far away from the rotocenter does it end up?

- If a point of the object starts out a distance d from the mirror line of a reflection, then how far away from the mirror line does it end up?

- Draw a line between a point and its reflected image point. What is the angle between this line and the mirror line?

We will say that a figure has rotational symmetry if ...

We will say that a figure has reflection symmetry if ...

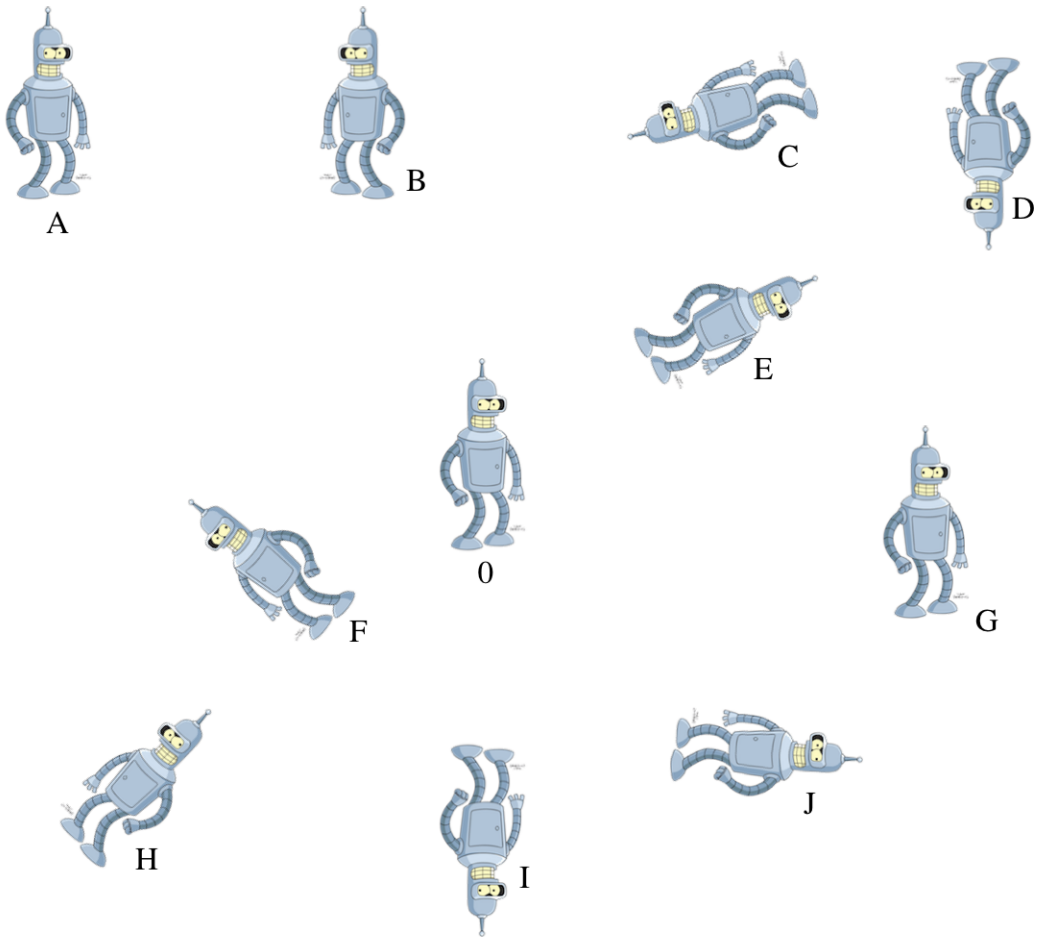
Etc.

We will say that a figure has (rigid motion) symmetry if:

Isometries in Geogebra

1. Make an account on Geogebra at www.geogebra.org
2. Go to the Geometry view (using top right menu).
3. Use the polygon tool at left to draw a polygon OR use the Media tool to upload an (asymetric) image OR use the Pen tool to draw a freehand shape.
4. Use the transformation tools to create a rotation, reflection, and translation of your original image.
5. Make one version in which you have labeled each transformed image by the type of isometry used to create it, and display the rotocenters, mirror lines, etc. used to make your transformations. This is your key.
6. Make another version in which only the original image is labeled, and all rotocenters, mirror lines, etc. are hidden. This is your puzzle sheet that you will challenge a classmate with. You can hide objects from the Edit tool "Show / Hide Object" or by selecting an object, selecting the three dots, then Settings, then deselecting the Show Object box.
7. Trade puzzle sheets with a classmate and see if they can figure out the isometry used to create each transformed image. Can you also figure out where the rotocenter is for the rotation, where the mirror line is for the reflection, etc?

Find the exact isometry



For each Bender A through G, determine what type of isometry will transform the original Bender (marked with a 0) to that Bender.

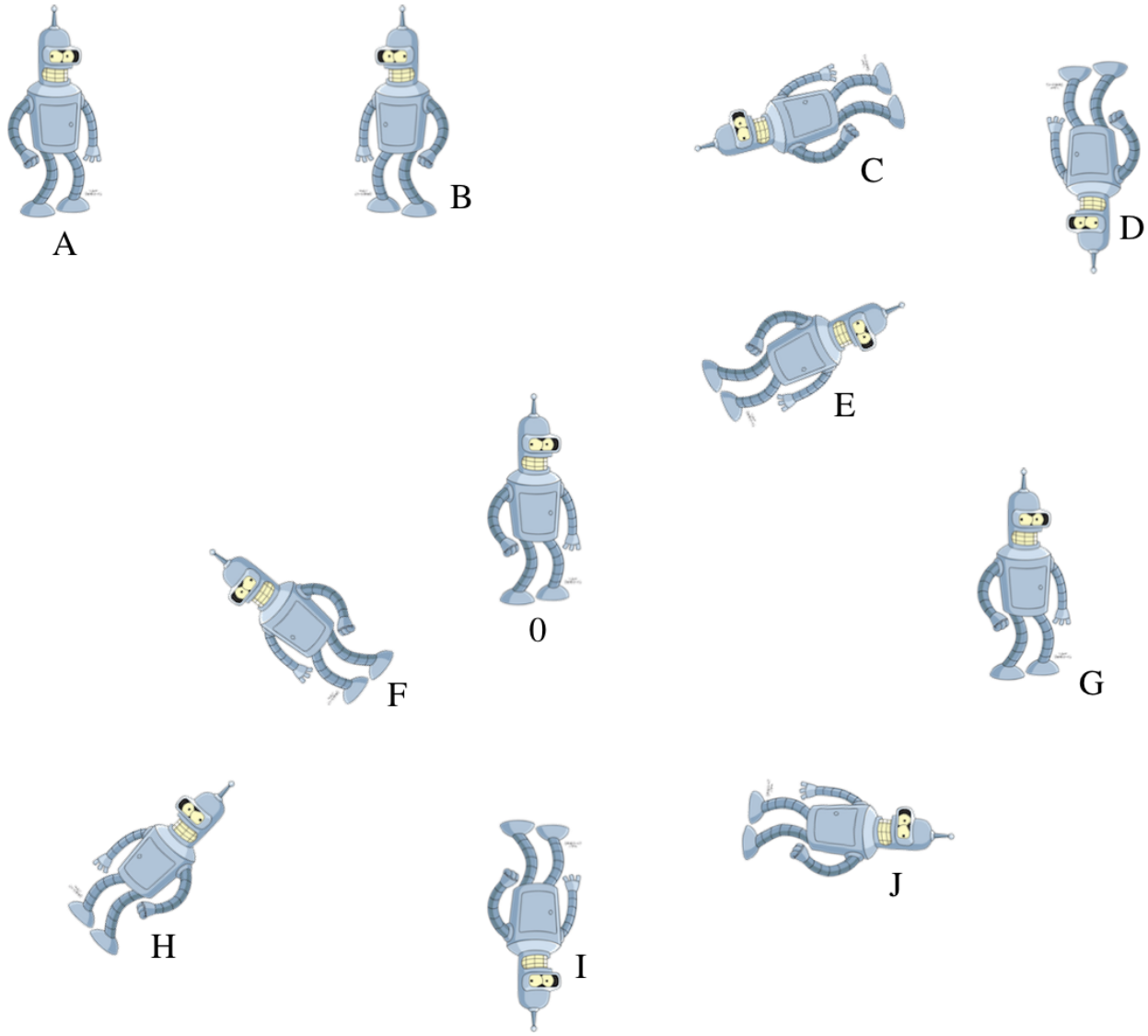
How can you tell what type of isometry is needed to transform one figure to its isometric image? Design a flow chart (decision tree) to systematically decide which type of isometry is needed.

Suggestion: to see if your flow chart is clear, ask a classmate to use it on some of the examples from class.

For each Bender A through G, determine which *specific version* of an isometry will transform the original Bender (marked with a 0) to that Bender. How can you determine these things *precisely*, not just as a rough estimate?

Give step by step instructions for each of the following:

1. how to find the translation vector for two images that you know are related by a translation.
2. how to find the mirror line for two images that you know are related by a reflection.
3. how to find the rotocenter AND the angle of rotation for two images that you know are related by a rotation.
4. how to find the glide line AND the translation vector for two images that you know are related by a glide reflection.


A
B
C
D
E
0
F
G
H
I
J

Hints:

- It is helpful to draw line segments connecting corresponding points in the original figure and its image.
- Think about midpoints and perpendicular bisectors.
- Start with examples where you already know the answer.
  - For example, start with a a glide reflection on Geogebra where you already know the mirror line.
  - Display the mirror line.
  - Then draw some line segments connecting corresponding points.
  - What is the relationship between these line segments and the mirror line?

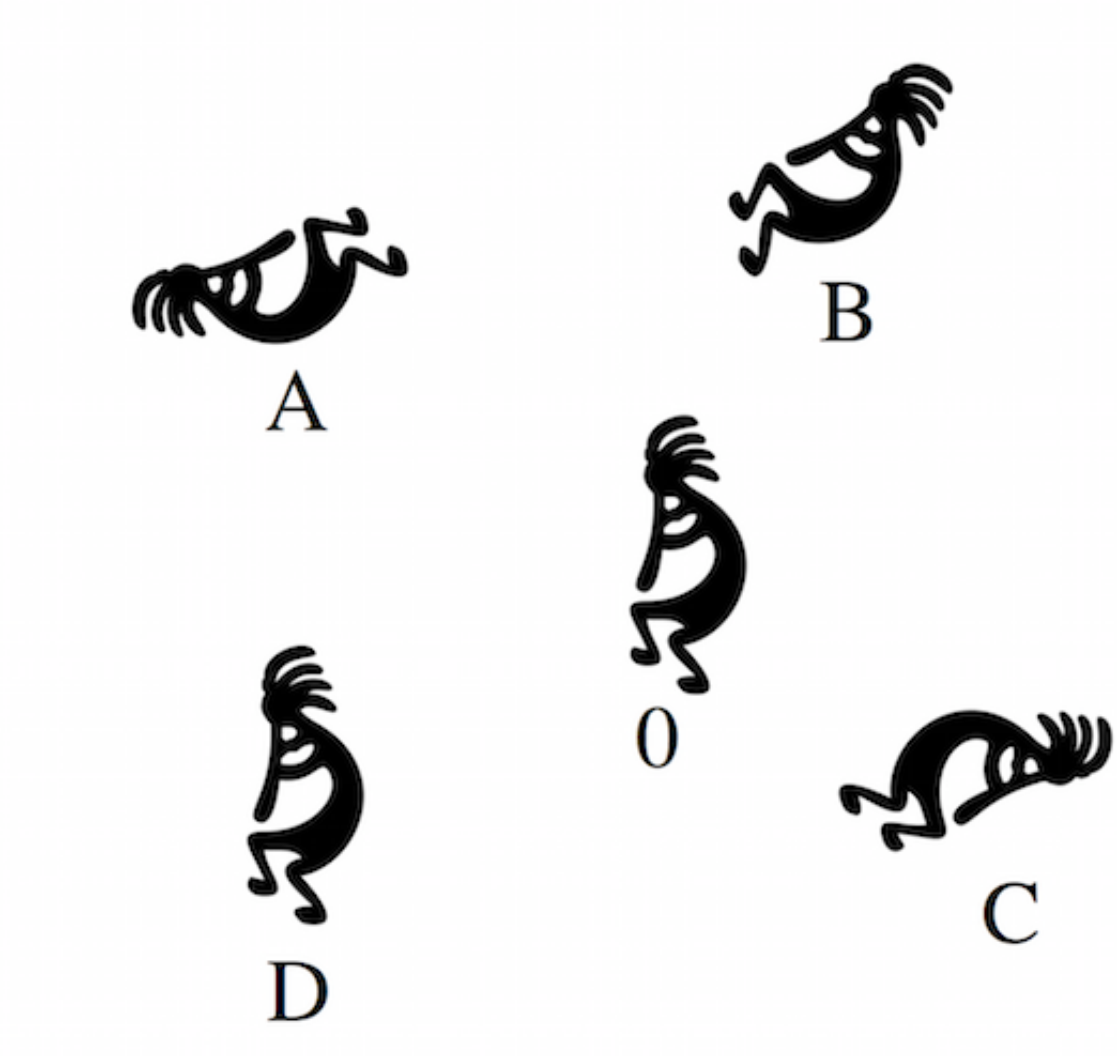
Note: you can use Geogebra to check your answers: first copy and paste the images into a new Geogebra file. Then use the polygon tool to trace over the original Bender and get a rough approximation. Then try applying a transformation to this approximation and see if the image lines up with another Bender.

Once you have found algorithms for finding

- the rotocenter of a rotation
- the mirror line for a reflection
- the glide line for a glide reflection

try to use geometry (similar triangles, etc.) to prove that the algorithms work.

Use your instructions to find the exact translation vectors, rotocenters and angle of rotations, mirror lines, and glide lines and translations to get from image 0 to images A, B, C, D in the picture of Kokopelli. You can use Geogebra to do this.



Find the exact translation vectors, rotocenters and angle of rotations, mirror lines, and glide lines and translation vectors to get from one dancer to the other in the pairs.