Final Projects

Students can work alone or in pairs on a final project. The project should include a powerpoint presentation and a written component. Some projects will also include a physical component, such as a mathematical work of art. For projects that include a physical component, the written component can be brief and may take the form of a description of how you made the project or a guide to using it. For projects without a physical component, the written component should take the form of a research paper. Final projects will be presented on the last day of class and during the final exam period on Dec. 5 from 12:00 – 3:00 pm.

Project Ideas

You are not limited to ideas on this list! Questions are intended to give examples of directions you could explore.

Maker Projects

1. New! **Paper models.**You might get some ideas from CutPutFoldUp.com. There are 5 pages of paper constructions including some 3-d tiles that tile 3-dimensional space.
2. **Projective Plane.** Use UNC's 3D printers to make models of the projective plane. The most common models are "Boy's Surface", the "Cross-Cap", and the "Roman" or "Steiner" surface. Dr. Goodman, professor emeritus at UNC, recently came up with a new model that she calls "Girl's Surface." There is a 3-d printer in the Kenan Library and one in the Hanes Art Center Maker space, or you can use other materials. If you want to use the Kenan Library 3D printer, here is a link to a form to submit 3D printing requests, and this is the email to set up a consultation to go over your file:KenanMakerspace@unc.edu . Mathematica can export directly to .stl format.
3. **Dodecahedral Tiles.** Use UNC's 3D printers to make a set of tiles based on a dodecahedron and an hourglass figure. Analyze the ways that these tiles can fit together to tile the plane. Here is one example of a periodic tiling with these shapes. Aperiodic tilings are also possible.
4. **Make Aperiodic 3D Tiles.** Use the 3D printers to make some aperiodic 3D tiles. There are some links to info on aperiodic 3D tiles here.
5. **Make a torus out of paper** See this article.
6. **Spherical symmetry in origami.** Build paper models of as many different types of spherical symmetry as possible. Check out cutOutFoldUp.com for some ideas. See also George Hart's constructions and slide together models
7. **Tiling patterns with origami.** Build origami patterns with frieze or wallpaper symmetry.
8. **Spherical symmetry Temari balls.** Embroider Temari balls or make models of spherical symmetry with patches of cloth covering foam balls, or use another method to make models of spherical symmetry.
9. **Spherical symmetry kaleidoscopes.** Kaleidoscopes with tapered sides make truly amazing spherical symmetry patterns. Here is a sample, some more samples and some videos (the Quilt Kaleidescope video has a picture of the set-up) . Page 63 of *The Symmetries of Things* has some guidelines for building them.
10. **Three-dimensional pattern kaleidoscopes.** *Symmetry, Shape, and Space: An Introduction to Mathematics Through Geometry* by Kinsey and Moore has information on a different kind of kaleidescope, that puts mirrors together in the pattern of a cube and other patterns to make infinite repeating figures in 3-dimensions.
11. **Hyperbolic quilt.** Sew a hyperbolic patchwork quilt. Even better: devise your own pattern!
12. **Hyperbolic tilings on the Poincare disk** # It is possible to use Geometer's sketchpad or other software to draw hyperbolic tilings similar to M. C. Escher's circle limit series and other artwork. See these instructions and this Geometer's Sketchpad file.
13. **Hinged Dissections.** Build animations or physical models of hinged dissections.
14. **Colorings of tilings.** For a few of your favorite Escher tessellations (or homemade tessellations, or other tessellations), demonstrate all possible ways of coloring the pattern in a symmetric way. See Chapters 11 and 12 of *The Symmetries of Things* for tables of 2-fold and 3-fold colorings.

15. **Coloring Rep-Tiles.** Systematically color some but not all of the sub-shapes of an aperiodic rep-tile, in the spirit of this coloring of Conway's pinwheel tiling to make quilt-like patterns with different characters. There are many rep-tiles that you could use instead of the pinwheel rep-tile.
16. **Musical frieze patterns.** Compose music that has frieze pattern symmetry: one motif is transformed and repeated using reflection, rotation, and translation. Musescore.org and Musescore.com and other software programs may be useful for experimentation. For physical models, music boxes like the one in the Vi Hart video Mobius Music Box can be purchased online for about $20.

Research Projects

1. New!**Soccer Ball History** Research the history of soccer ball patterns. When did the standard black and white pattern become popular? What are the all the official world cup patterns and what symmetry types do they have? Are there any symmetry types that are not used for World Cup balls, and if so, which and why might they not be used? Etc.
2. New!**Map Projection** Since the earth is curved, there is no way to make a flat map without cutting gaps or stretching and distorting some parts. Research the types of maps that can be made to represent the earth and the pros and cons of each. The same thing can be done for hyperbolic plane, instead or in addition.
3. **Escher Exhibit.** Tour the exhibit at the Raleigh museum of art. Build a project around your tour. For example, you could list all works on display, and categorize them by which ones involve wallpaper patterns, other symmetry patterns, or no symmetry patterns, and sub-categorize the ones involving wallpaper patterns by which of the 17 types is represented. Or, describe the styles and mathematical themes used in the works on display and analyze which themes occur during which time periods of Escher's work. Note: photography is not allowed in the exhibit.
4. **Study wallpaper or frieze symmetry patterns used in a particular style of art.**, for example, 19th century American quilting patterns, or Islamic lattice patterns, or Chinese pottery from a particular era, or Native American basket weaving. Are all possible symmetry patterns represented? If not, can you come up with a theory for why certain patterns are absent? Are some patterns more common than others? Why? (See section 5.5 in Symmetry, Shape, and Space book by Kinsey and Moore for Islamic lattice patterns.)
5. **Escher.** Analyze the wallpaper patterns used in Escher's tessellating patterns. Does he use all 17? Does he use some types more frequently during different periods of his life?
6. **Analyze frieze patterns used in architecture.** Are all 7 types commonly used? Are some types used more in some types of architecture?
7. **Frieze patterns in clothing.** Are all 7 types equally represented?
8. **Wallpaper patterns in clothing.** Are some pattern types used more in men's clothes (e.g. neck ties) and other types used more in women's clothes (e.g. dresses or scarves)?
9. **Frieze patterns in music.** What types of frieze patterns occur in what styles of music? Crab Cannon by Bach is one famous example. See this article by Vi Hart and this article by Alissa Crans et al. for more ideas.
10. **Symmetry in biology.** Which types of organisms have finite symmetry patterns and which don't? Is symmetry type something that tends to be the same for related species or vary among related species? What types of organisms show frieze pattern symmetry? Wallpaper symmetry? Fractal symmetry?
11. **Double Strip Patterns.** Report on the types of double strip patterns analyzed in this paper and convert the signatures to orbifold notation.
12. **Three dimensional isometries.** In the 2-dimensional plane, we only needed four types of motions to describe all isometries: reflection, rotation, translation, and glide reflection. Are all these motions possible in 3-dimenstions? Are their other isometries possible in 3-d? What happens when you combine isometries in 3-d? Is a reflction of a reflection still always a rotation or translation? Etc. What about isometries in 4-dimensions?!
13. **Crystallographic patterns (three dimensional wallpaper patterns).** See *The Symmetry of Things*, Chapters 22 and 25. How many are there? Do they all actually occur in real crystals? What

notatation can be used to describe and categorize them? Do they have other appearances in art and nature besides rock crystals?

14. **Three-dimensional tilings with polyhedra.** It is easy to tile the plane (2-dimensions) using triangles, squares, or hexagons, and also possible to tile it using pentagons (but not regular pentagons. What about tiling three dimensional space with polyhedra? Cubes definitely work. What other shapes can tile three-dimensional space? Here is one example that you can make out of paper.
15. **Tiling shapes with dominos.** Which can be tiled? Coloring arguments. Tiling triangles with three-triangle tiles. Tiling rectangles with ell-shaped tiles. Etc. See this Julia Robinson handout for some ideas.
16. **Aperiodic 2-dimensional tiles.** Aperiodic tiles are tiles that can tile the plane, but never in a way that that has translation symmetry. We will cover this topic only very lightly in class, so this would be a good project if you are interested in learning more.
17. **Aperiodic Decagons.** Investigate the **aperiodic** tilings that can be made from a decagons and hourglass shapes. Here is a periodic tiling that can be made from these shapes.
18. **Aperiodic 3-dimensional tiles.** Aperiodic tiles in 2-dimensions are tiles that can tile the plane, but never in a way that that has translation symmetry. Do aperiodic 3-dimensional tiles exist? If so, what are some examples?
19. **Quasicrystals.** are aperiodic tilings that occur in nature, as crystals. That is, they have an ordered appearance that could be extended forever, but no translation symmetry. Here is a description on wikipedia.
20. **Groups.** In math, a group is a bunch of things (the "group") and a way of combining pairs of things. The symmetries of a finite figure can be described as a group, because when you combine two symmetries by following one by the other you get another symmetry of the finite figure. See sections 3.2 through 3.4 and chapter 6 of Groups And Symmetry: A Guide to Discovering Mathematics by David Farmer.
21. **Explain alternative notations for symmetry patterns.** How can the orbifold notation that we use for wallpaper patterns be used to describe frieze patterns? What other notations are commonly used for frieze patterns and wallpaper patterns, what is their history, and how do they work? What are the advantages of one over system over the other?
22. **Fractal symmetry.** Your report could focus on any of the following: history, applications, fractal dimension.