

# Part X

## Frieze Patterns Introduction

The goal for this part is to identify the symmetries in frieze patterns.

# Rosettes and Finite Figures revisited

Which of the following statements are true?

1. If a figure is a rosette, then it has finitely many symmetries .
2. If a figure has finitely many symmetries, then it is a rosette.
3. If a figure is a rosette, then it has no translation symmetries.
4. If a figure has no translation symmetries, then it is a rosette.
5. If a figure has finitely many symmetries, then it has no translation symmetries.
6. If a figure has no translations symmetries, then it has finitely many symmetries.

# What are frieze patterns?

A *frieze pattern* is a pattern that has translation symmetries in (only) one direction.



They are also called *strip patterns* and *border patterns*. We will usually assume our strip pattern is oriented so that translations are in the left right direction. That is, the center line of the strip is horizontal.

# Examples of frieze patterns



## Mark the symmetries of frieze patterns

For each of the following strip patterns, mark all distinct symmetries as follows:

1. Mark the smallest translation in the left to right direction with an arrow.
2. Draw mirror lines with solid lines.
3. Mark rotocenters with circles and numbers indicating the order of the rotation.
4. Mark the (smallest) translation vectors for glide reflections with dotted line arrows.

For any frieze pattern, a **fundamental domain** is a piece of the pattern of the smallest possible area that can be repeated to make the entire pattern, using isometries that are symmetries of the entire pattern. For each of the frieze patterns above, identify a possible fundamental domain and mark it. (Suggestion: as a first step, look for a translational symmetry and reduce from there. )

# Make your own frieze patterns

For each of the frieze patterns examples, replicate the pattern using a footprint motif. That is, put a footprint in the fundamental domain and make a frieze pattern with the exact same symmetries as the given pattern.

# Frieze pattern practice

Ignore minor color differences and other minor asymmetries.



From the Mathematical Association of America.

Mosaic  
Nuestra Señora de la Almudena  
Madrid, Spain

Tile Frieze  
Palacio de Velazquez  
Parque de Retiro  
Madrid, Spain

Back of a Bench  
Banos de la Maria de Padilla Reales  
Alcazares  
Seville, Spain

Ceiling  
Mezquita  
Cordoba, Spain

Mosaic Border  
Alcazar de los Reyes Cristianos  
Cordoba, Spain

Meander Frieze  
San Giorgio Maggiore  
Venice, Italy

# Homework

1. For each pattern in the collection of European frieze patterns, do the following:
  - 1.1 Mark a fundamental domain by drawing a line around it.
  - 1.2 List all the types of symmetries that it has, using the following letters: H for reflection through a horizontal mirror line, V for reflection through a vertical mirror line, R for rotation, G for glide reflection, and T for translation.
2. Draw three different frieze patterns, either using Geometer's Sketchpad, or stamps and a stamp pad, or paper and pencil, or paint, etc. Try to make your examples different from each other by using different motifs AND by incorporating different types of symmetry.