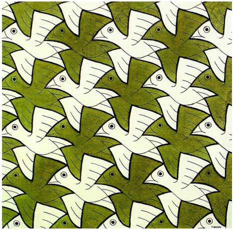
Part XV

Drawing wallpaper patterns

The goal for this part is to draw Esher-style wallpaper patterns with interlocking figures.

Translation only ◦

What shape can be used to make a simple fundamental domain for a translation only pattern?



- In Geometer's Sketchpad, start with that shape, but build two adjacent edges out of jagged line segments and arcs.
- Then apply translations to tesselate the plane.
- Go back to the original motif and drag the line segments and arcs to modify the shape.
- Try to make your motif look like an animal or other object.

Glide and reflection $\times *$

Use the bulldog template posted on Sakai to build a wallpaper pattern with $\times *$ symmetry.

- Hint: you will need to create a vertical mirror line to do the glide reflection.
- It is handy to glide reflect a single point and then "Define a Custom Transform" in order to apply repeated glide reflections without creating intermediate figures.

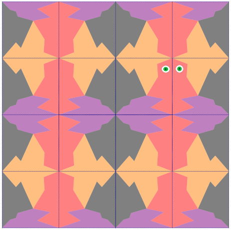
Kaleidoscope patterns $*333$

What shape makes a simple fundamental domain for a $*333$ wallpaper pattern?

- Try to make a $*333$ pattern that tessellates the plane with interlocking figures.
- You can use Prof. McCombs Fish-Lizard-Bat template on Sakai.
- Or, you can make your own by starting with an equilateral triangle and filling in 3 pieces of it with 3 different colored polygons.

Kaleidoscope patterns $*2222$

What shape makes a simple fundamental domain for a $*2222$ wallpaper pattern?

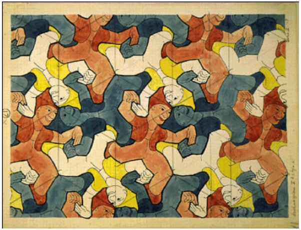


Make a $*2222$ pattern that tessellates the plane with interlocking figures.

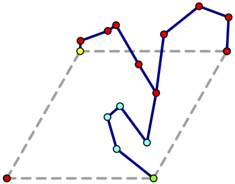
- Start with the fundamental domain.
- Divide it up into polygons of different colors.
- Reflect along the edges repeatedly.
- Push and pull your polygon vertices to change the shape of your figures.

Gyroscope patterns 333

What shape makes a simple fundamental domain for a 333 wallpaper pattern?



Start with the fundamental domain shape and change the straight line edges to jagged lines and arcs.



Homework

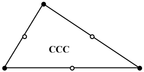
1. Read Chapter 3 in the textbook if you have not already. Optional and ungraded but recommended: do the 4 exercises at the end of Chapter 3. Answers are given on the opposite pages.
2. Design an Escher-style wallpaper pattern with interlocking figures that fill the paper. You can use Geometer's sketchpad, other software, or draw one by hand. Please identify the signature of the pattern. For full credit, your pattern needs to have recognizable shapes that represent animals or other objects.

You can read about Conway's criteria for building interlocking wallpaper patterns with rotational symmetry in the following document from http://www.mathforamerica.org.

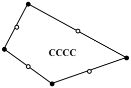
The five types of Conway Criterion polygon tile

In each polygon, black dots are vertices while small circles denote midpoints of edges. Although the polygons shown are convex, all except the triangle can be nonconvex.

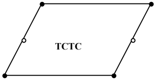
In each of these tiles, the straight edges can be replaced by arbitrary curves *that have the required symmetry*. The notation given is that of H. Heesch, and indicates which edges have halfturn symmetry (C means centrosymmetric--the edge can rotate onto itself by a halfturn about the midpoint) and which are related to another by a translation (T).



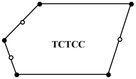
Three edges, each with halfturn center



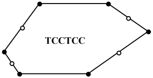
Four edges, each with halfturn center



Four edges, two with halfturn centers, two related by translation



Five edges, three with halfturn centers, two related by translation



Six edges,four with halfturn centers, two related by translation

Exercise in using the Conway criterion

Make an example of each of the five possible Conway tiles.
A polygon of each type is on the other side of this page.

Your tiles should be more interesting than polygons, replacing the straight edges with curved or ragged edges, to produce an Escher-like tile, or a decorative tile with curved edges. First, choose the type of tile you want to make, and on your paper (or sketch window) choose the positions of the vertices of your tile. Lightly draw the straight edges that connect those vertices to make a polygon. Mark the midpoints of any edges that are to be centrosymmetric.

Now replace the straight edges of your polygon. An edge that is to be translated can be any curve that joins its two endpoints (provided that when that curve is translated to a matching edge, it doesn't intersect itself). Translate this curved edge to produce a matching edge that joins two other vertices. An edge that has halfturn symmetry is created by joining one endpoint to the midpoint with any curve, then rotating that curve 180° about the midpoint so that it connects the midpoint to the other endpoint. Each edge that has halfturn symmetry can be independently produced by this method.

Use method (a) or (b).

(a) Use graph paper to guide the drawing of the boundary of your tile. You can also make a template of each of the free boundary portions of your tile so that you can turn these portions by 180˚ or translate them to produce the related portions of the boundary of your tile.

(b) Use *The Geometer's Sketchpad* or other interactive program to make tiles using transformations to translate and rotate the free portions of the boundary of your tile.

When you have a satisfactory tile, make a patch of tiling by fitting together copies of the tile, using 180° rotations about the centers at midpoints of tile edges or translations that match opposite edges. Your patch should show at least one tile completely surrounded by others. You will find that half your tiles are "upside down"; this is a result of the halfturns. Finally, color your tiling so that adjacent tiles have different colors. If you use Escher's coloring rule, you will use the fewest number of colors possible to do this. In most cases, you can use just two colors.

When making tilings by hand, you can repeat a tile by carefully redrawing the tile outline guided by the graph paper, or by tracing around a template of the tile, or by using tracing paper. Or you can photocopy your tile several times and cut out the copies and fit them together to make a patch of tiling. Or you can make a template to trace and cut out copies of your tile from different colors of paper and then fit together the tiles and glue them down.

For more information on the Conway Criterion, see "Will it tile? Try the Conway Criterion!" Doris Schattschneider, *Mathematics Magazine*, vol. 53, no. 4, September 1980, pp. 224-233.

For information on *The Geometer's Sketchpad*, go to *http://www.dynamicgeometry.com/*

To see all of Escher's tessellations, and learn how he made them, see the book *M.C. Escher: Visions of Symmetry*, by Doris Schattschneider, published by Harry Abrams, 2004.