Part XVII

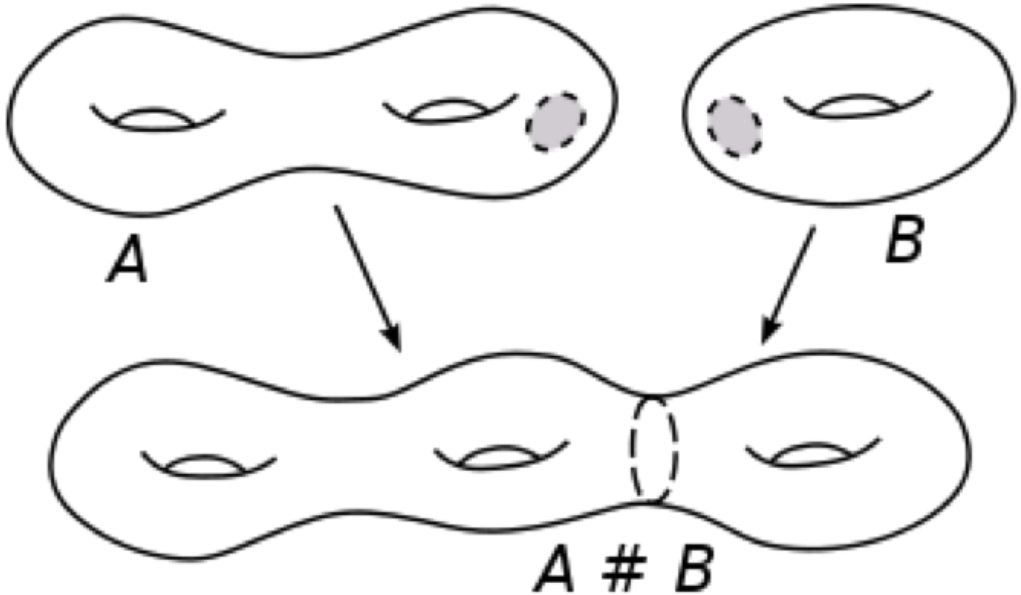
Classification of Surfaces

The goal of this section is to classify topological surfaces that are finite and have no boundary.

Some of this material is taken from *The Shape of Space* by Jeff Weeks.

Making new surfaces out of old

The connected sum of two surfaces is what you get when you remove a disk from each surface and attach them together along the gaping boundaries.



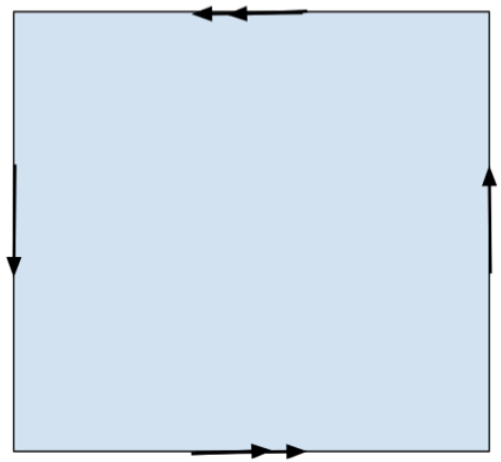
Connected summing stuff

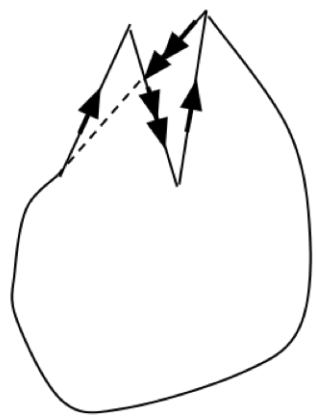
- What do you get when you take the connected sum of a torus and a sphere? A Klein bottle and a sphere?

- What do you get when you take the connected sum of two tori?

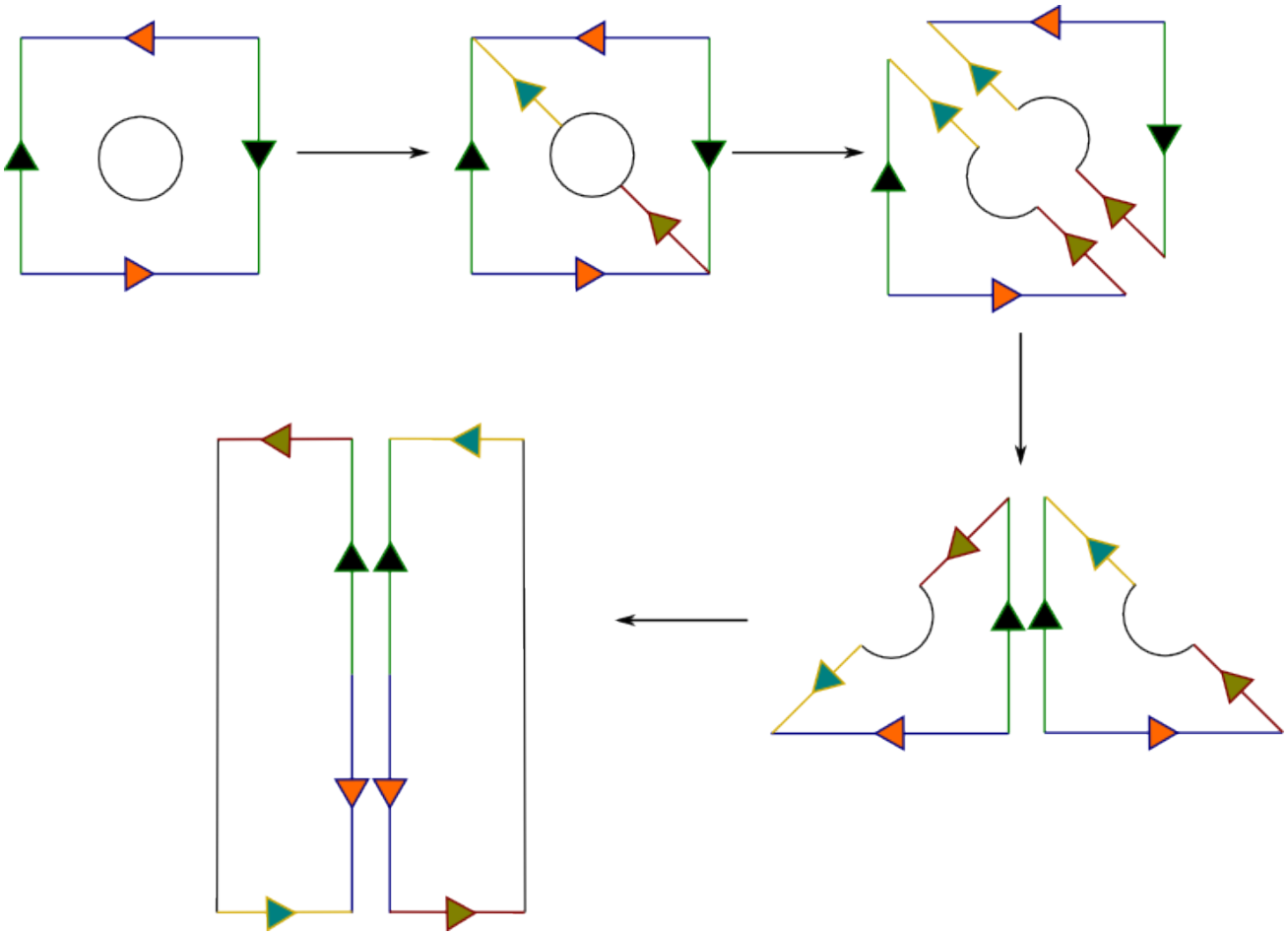
- What do you get when you take the connected sum of two projective planes?
  Hint: First, try to figure out what surface you get when you remove a disk from a projective plane.

Remember the Projective Plane





Removing a disk from a projective plane



From http://www.math.cornell.edu/ mec/

The connected sum of two projective planes

To form the connected sum of two projective planes,

- First remove a disk from each of them.
- This gives us two _____
- Now glue together the two boundary circles
- This gives us _____

Building all surfaces

Theorem (Classification of Surfaces) Every (finite) surface without boundary can be built by connected summing tori and projective planes onto a sphere.

# PP / # tori	0	1	2	3	4
0					
1					
2					
3					
4					

Find a surface in the table that is equivalent to

- $K^2 \# P^2$
- $K^2 \# T^2$
- $K^2 \# K^2$

Duplicates

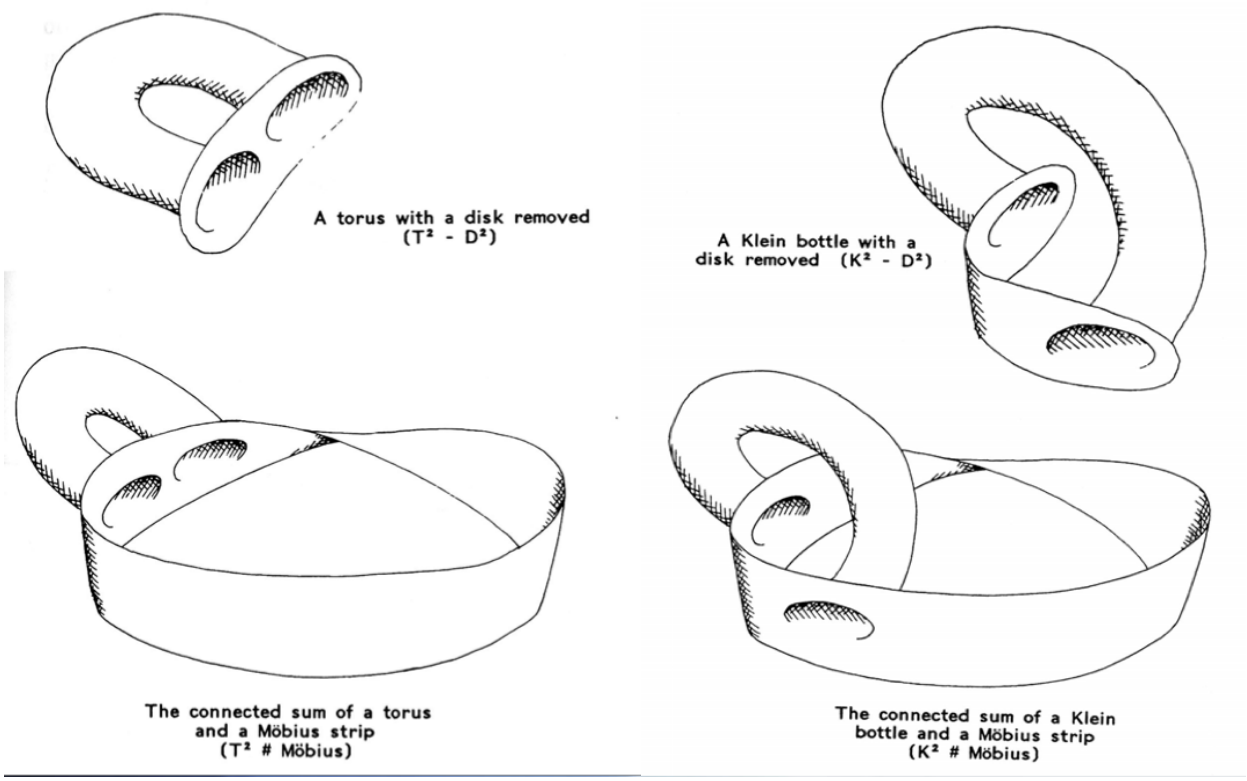
Are there any duplicates in this table?

# PP / # tori	0	1	2	3	4
0					
1					
2					
3					
4					

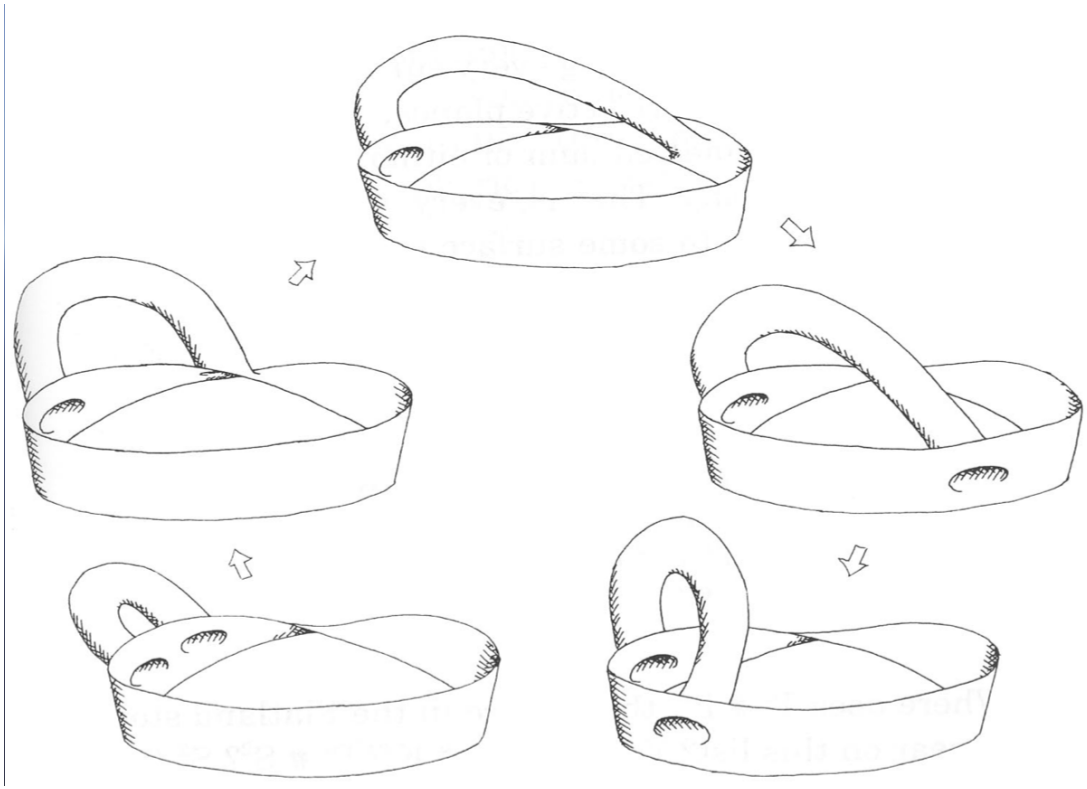
Consider the two surfaces $T^2\#P^2$ and $K^2\#P^2 = P^2\#P^2\#P^2$

Connected sum with Mobius band

Study these pictures from Jeff Week's book *The Shape of Space*



Deforming a handle to a cross-handle



How does this show that $T^2 \# P^2 = K^2 \# P^2$?

Reduced list of surfaces

# PP / # tori	0	1	2	3	4
0					
1					
2					
3					
4					

S^2

T^2
$T^2 \# T^2$
$T^2 \# T^2 \# T^2$
$T^2 \# T^2 \# T^2 \# T^2$
$\dots$

P^2
$P^2 \# P^2$
$P^2 \# P^2 \# P^2$
$P^2 \# P^2 \# P^2 \# P^2$
$\dots$

Match the surfaces

Match the surface from the left column with the equivalent surface from the right column.

Column A	**Column B**
$S^2 \# T^2$	$P^2 \# P^2$
K^2	$K^2 \# P^2$
$S^2 \# S^2 \# S^2$	$P^2 \# P^2 \# P^2 \# K^2$
$P^2 \# T^2$	$S^2 \# S^2$
$K^2 \# T^2 \# P^2$	T^2

From The Shape of Space by Jeff Weeks