Part XXVI

Hyperbolic Geometry

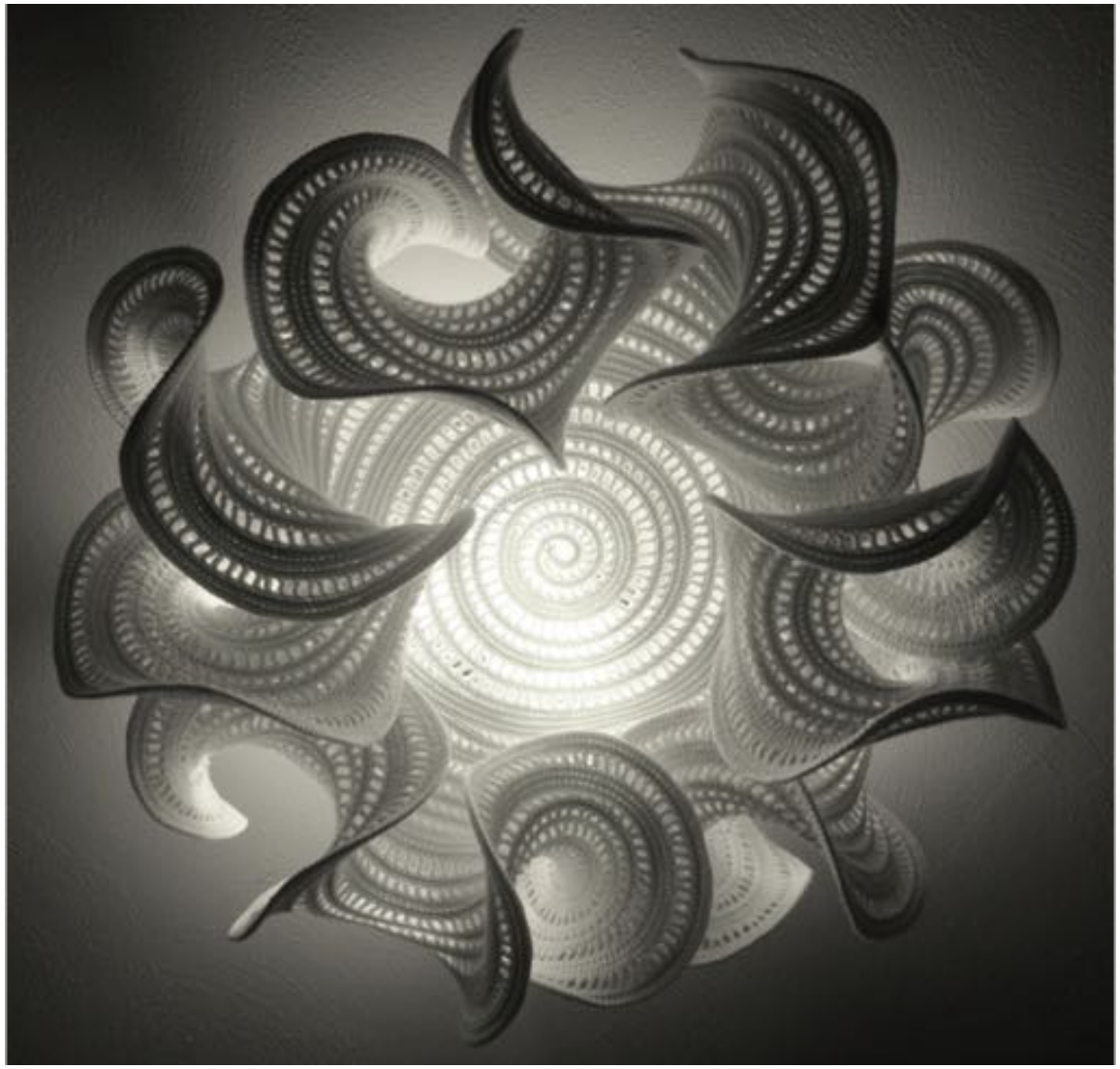
The goal for this part is to become more familiar with negatively curved surfaces by making hyperbolic soccer balls.
Ideas, templates, and instructions are from Frank Sottile
`http://www.math.tamu.edu/~sottile/research/stories/hyperbolic_football/`

Negatively curved surfaces in nature

Negatively curved surfaces in art

by Daina Taimina

Negatively curved surfaces in art



by Gabriele Meyer

Negatively curved surfaces in art

by Gabriele Meyer

Negatively curved soccerball patterns

Recall that a standard soccerball has one black pentagon and two white hexagons at each vertex.

The angle defect at each vertex is:

What is the angle defect if we use one black heptagon (7-sided polygon) and two white hexagons at each vertex?

Hyperbolic soccerballs

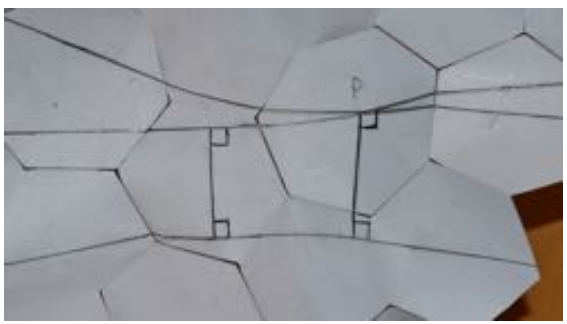
Use the templates to tape together one black heptagon and two white hexagons at each vertex.
Hints:

- Do as little cutting as possible. Don't separate all the polygons. Instead cut around the outside of a patch of hexagons, and separate only when necessary to insert a heptagon or more hexagons.
- Tape together along edges only. Don't try taping over vertices, where there is curvature!
- It's easier to work with a partner.

The geometry of negatively curved spaces is called hyperbolic geometry

In (flat) Euclidean geometry, given any straight line, and any point not on the line, there is exactly one parallel line through that point. What is the nature of parallel lines on a negatively curved surface?

1. To draw a line on the model, flatten part of the model to start the line, and use a short (< 15 cm) straightedge to continue the line across the model, flattening pairs of polygons as needed. Try to avoid running the line through a vertex.
2. After drawing your line completely across the model, you can pick it up, straighten it along the line, and sight down the line to see that it is straight.
3. After drawing a first line, pick a point on it and draw a short line segment m perpendicular to it. Then start a new line perpendicular to m and extend this third line across the model. What do you notice about your original line and this new, parallel line?
4. On one of the parallel lines from the previous step, choose a point P not lying on the common perpendicular m. Dropping a perpendicular from P to the other line, and then taking a perpendicular to that through P gives a second line through P that is parallel to the original line.



Hyperbolic triangles

1. Now, try to draw a triangle. For this it is best to try to make a big triangle.
2. Measure its interior angles.
   - One method is to mark off an arc on a small sector of a circle (cut out of a scrap of paper), lay the semicircle on a flat surface, and then use your protractor
   - It is possible to find the sum of the three angles by marking off three consecutive arcs along a small sector of a circle.
3. What is the sum of the interior angles of the triangle?
4. How far does the sum deviate from 180°?
5. Compare the deviation of the sum from 180° to the total angle defect from the interior vertices.
6. Explain this using Gauss-Bonnet.
7. Draw a triangle that encloses as much curvature as possible.