

## Part II

# Isometries of the Plane

The goal of this section is to identify isometries.

## Geometer's Sketchpad

Use Geometer's Sketchpad to experiment with rotations:

- ▶ Use the Polygon tool on the left to draw any figure.
- ▶ Then use the Point tool to draw an arbitrary point (on or off the figure).
- ▶ Next use the Selection tool (the arrow) and double click the point to select it as a center of rotation (or you can use the Transform menu and Mark as Center).
- ▶ Select the polygon and use Transform > Rotate to rotate by some angle of your choosing about the rotocenter.
- ▶ Experiment by changing the angle and moving the rotocenter.

Draw a figure that looks exactly the same after rotating. (In other words, draw a figure that you could apply a rotation to, while a friend closed their eyes, and the friend wouldn't be able to tell the difference.)

Use Sketchpad to draw each of the other types of isometries. For each type of isometry, decide if it is possible to draw a figure that remains unchanged after applying the isometry.

# Lots of Benders

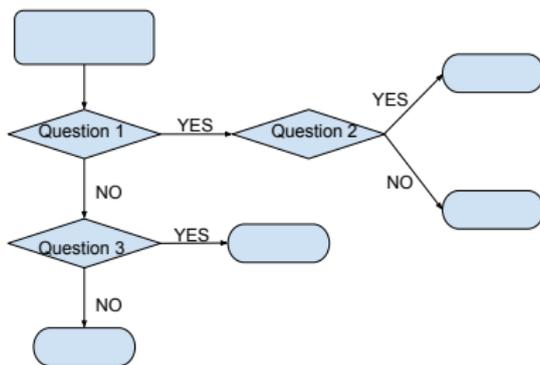
For each Bender A through G, determine what type of isometry will transform the original Bender (marked with a 0) to that Bender.



Note: you can use Sketchpad to experiment and test your answers: first copy and paste the images on the handout into a new Sketchpad file. Then use the polygon tool to trace over the original Bender and get a rough approximation. Then try applying a transformation to this approximation and see if the image lines up with another Bender.

# Identifying isometries

- ▶ How can you tell what type of isometry is needed to transform one figure to its isometric image? Design a flow chart to systematically decide which type of isometry is needed.



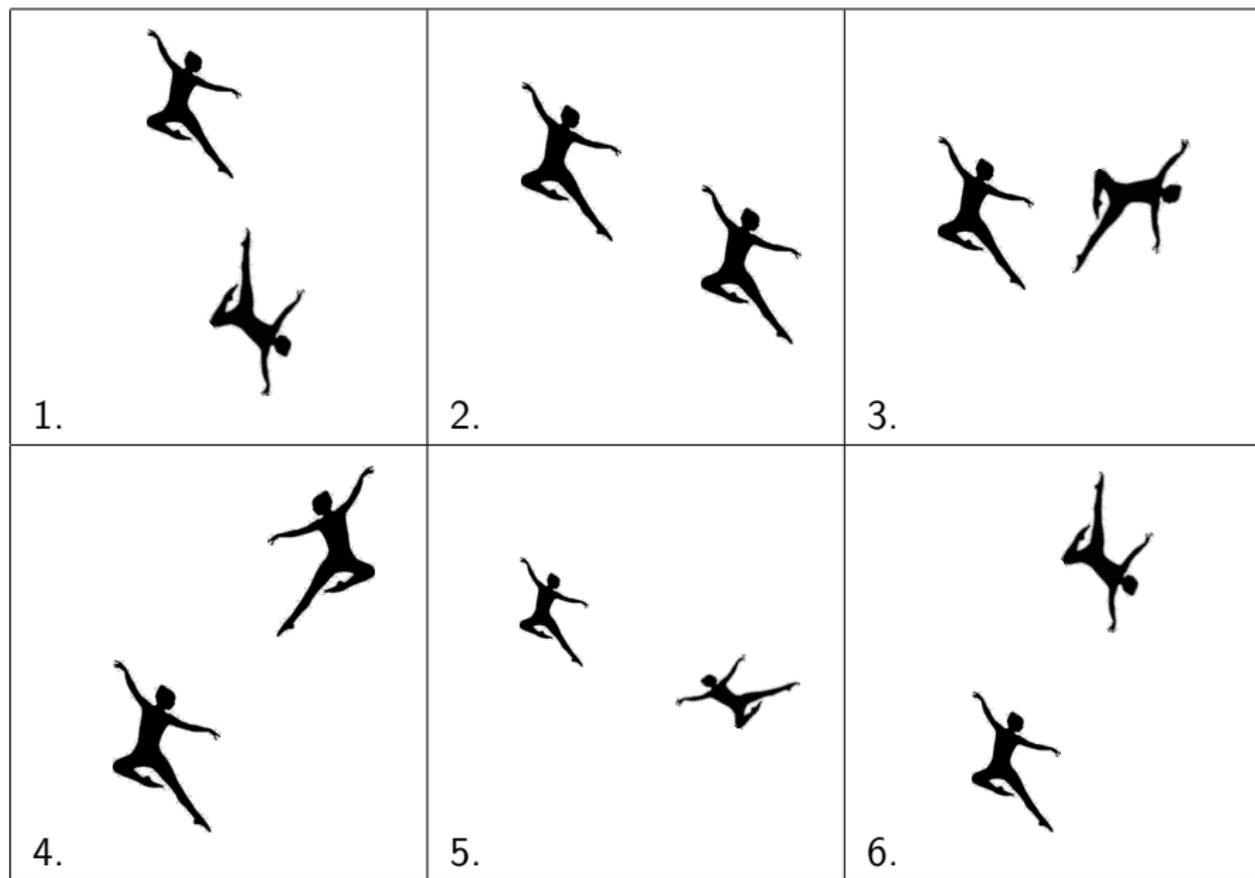
# Identifying isometries

For each pair of dancers, determine which type of isometry is needed, and also which specific version of this isometry is needed:

- ▶ If a translation is needed, what translation vector is needed?
- ▶ If a reflection is needed, where is the mirror line?
- ▶ If a rotation is needed, where is the rotocenter and what is the angle of rotation?
- ▶ If a glide reflection is needed, where is the mirror line and how far the the translation?

How can you determine these things *precisely*, not just as a rough estimate? Give clear, step-by-step instructions that you can apply to other examples, like the dinosaur stamp symmetry page.

# Pairs of dancers



# Hints for identifying isometries

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- ▶ It is helpful to draw line segments connecting corresponding points in the original figure and its image.
- ▶ Think about midpoints and perpendicular bisectors.
- ▶ Start with examples where you already know the answer.
  - ▶ For example, start with a glide reflection on Geometer's Sketchpad where you already know the mirror line.
  - ▶ Display the mirror line.
  - ▶ Then draw some line segments connecting corresponding points.
  - ▶ What is the relationship between these line segments and the mirror line?

# Dinosaur stamp symmetry



A



B



C

# Kokopeli



A



B



O



## Homework

1. Make a flow chart (or decision tree) that someone could use to decide what kind of isometry (reflection, rotation, translation, glide reflection) is needed to bring one figure to its image. (Suggestion: to see if your flow chart is clear, ask a friend to use it on some of the dancer or other examples from class.)
2. Design your own "stamp symmetry" page using Geometer's sketchpad, like the "Lots of Benders" one. To do this, copy and paste any image onto Geometer's Sketchpad and label it as "0" using the Text Tool at left. Then perform at least 4 different isometries on this image and label them A, B, C, D. Then use Display > Hide to hide the points and lines that you used to make the isometries. Print this out using the Print Preview > Fit to Page so that it will all be on one page, and bring it to class for a classmate to test their flow charts on. Please also turn in a copy with your homework.
3. Write down what type of isometry relates each the 6 pairs of dancers. You do not need to specify any details, just write "translation" or "reflection" etc.

## More Homework

1. Give step by step instructions for each of the following:
  - 1.1 how to find the translation vector for two images that you know are related by a translation.
  - 1.2 how to find the mirror line for two images that you know are related by a reflection.
  - 1.3 how to find the rotocenter AND the angle of rotation for two images that you know are related by a rotation.
  - 1.4 how to find the glide line AND the translation vector for two images that you know are related by a glide reflection.
2. Use your instructions to find the exact translation vectors, rotocenters and angle of rotations, mirror lines, and glide lines and translations to get from image 0 to images A, B, C, D in the picture of Kokopelli.