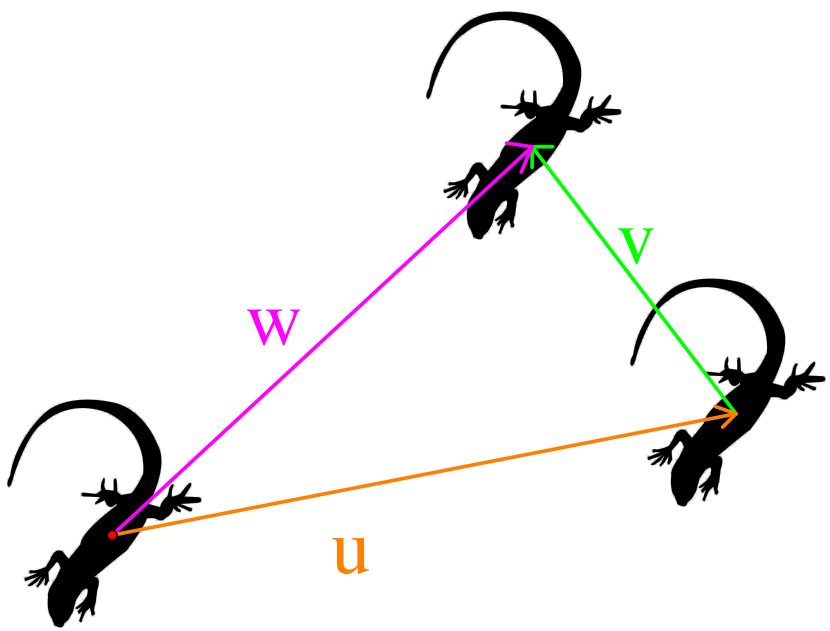
Part V

Combining Isometries

The goal for this lesson is to figure out what happens when we combine isometries; that is, do one isometry followed by another.

Translations of translations

- Suppose we do a translation using vector u, and then do a translation using vector v. We could accomplish the same final result using a single translation through the vector w. So we say that a translation followed by a translation is a translation.



Combining isometries chart

Fill out the following chart. Do the isometry on the left first, then the isometry on the top.

	Translation	Rotation	Reflection	Glide
Translation				
Rotation				
Reflection				
Glide				

Be careful: most squares will have more than one answer. For example, a reflection of a reflection is a rotation when the mirror lines intersect and a translation if the mirror lines are parallel.

Symmetry group of a square

The square has 4 mirror lines and 4 rotations (if we count the do nothing as a rotation by 0°). Using the following notation for the isometries of the square, fill in the white squares of the following chart for what happens when you follow the isometry listed at left by the isometry listed at top.
Hint: Think about whether you get a rotation or a reflection. Then follow what happens to a single point, like point A, to determine which rotation or which reflection.

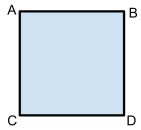
Multiplication Table for the Symmetries of a Square

SYMMETRY GROUP OF A SQUARE

The "symmetry group" of a square has 4 reflections and 4 rotations (if we count the do nothing as a rotation by 0°). Using the following notation for the isometries that are symmetries of the square, fill in the chart for what happens when you follow the isometry listed at left by the isometry listed at top. The first two lines are filled in for you.

	e	r_{90}	r_{180}	r_{270}	f_{-}	$f_{\vert}$	$f_{\backslash}$	$f_{/}$
e	e	r_{90}	r_{180}	r_{270}	f_{-}	$f_{\vert}$	$f_{\backslash}$	$f_{/}$
r_{90}	r_{90}	r_{180}	r_{270}	e	$f_{\backslash}$	$f_{/}$	$f_{\vert}$	f_{-}
r_{180}								
r_{270}								
f_{-}								
$f_{\vert}$								
$f_{\backslash}$								
$f_{/}$								

Hint: Think about whether you get a rotation or a reflection. Then follow what happens to a vertex, like point A, to determine which rotation or which reflection.



- e = do nothing
- r_{90} = rotation by 90° counterclockwise around center of square
- r_{180} = rotation by 180° counterclockwise around center of square
- r_{270} = rotation by 270° counterclockwise around center of square
- f_{-} = flip across a horizontal mirror
- $f_{\vert}$ = flip across a vertical mirror
- $f_{\backslash}$ = flip across diagonal mirror line [1]

Homework

1. Fill out the multiplication table for the 8 isometries that are symmetries of the square. The"product" of two isometries means you follow one by the other. For example, the product of reflection across a horizontal line and rotation by 90 degrees counterclockwise is reflection through a diagonal line that extends NorthEast to SouthWest.
2. Write the isometry that takes external link: dinosaur A to dinosaur C as a product of three reflections. Draw the three mirror lines involved and specify what order to apply the reflections. (Hint: use your algorithm from the last homework.)
3. Sometimes a rotation followed by a rotation is a translation, not a rotation. When does this happen? Explain your reasoning.