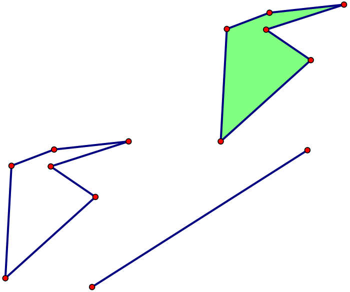
Part VI

Order, Inverses, and Commutatitivity

The goal for this lesson is to figure out what happens when we repeat isometries, do isometries in the opposite order, or try to undo isometries.

Inverses

The inverse of an isometry undoes the action of the isometry.



Experiment with inverses

1. Draw an irregular polygon and act on it with a translation. Fill the interior of the image triangle. Now find an isometry that will plop this interior back inside the original triangle. This isometry is the inverse of your original translation.
2. Repeat this process with other types of isometries.
3. Does every isometry have an inverse? If not, find an example of one that does not.

Conclusions about inverses

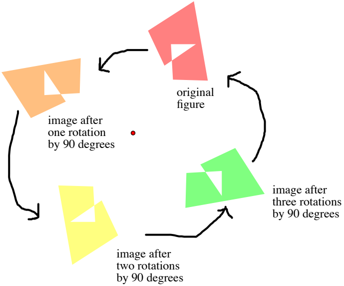
Fill out the following chart:

Isometry	Inverse
Translation in the direction of vector $\vec{v}$	
Rotation by angle θ in the counterclockwise direction	
Reflection across mirror line m	
Glide across mirror m and along vector $\vec{v}$	

Order

The order of an isometry is the number of times you have to repeat it to get back to what you started with. Order is called infinite if you never get back to what you started with.

For example, a 90° rotation has order 4, because when we perform it on a figure four times consecutively, we get back to where we started. Four times consecutively means first perform it on the original figure, then on the resulting image, then on that image, etc.



Experiment with order

- Draw an irregular polygon and act on it with some isometry.
- Fill the interior of the image polygon. Now repeat the action of the same isometry on this interior.
- Do this several times, always using the same isometry and acting on the newest image, to find the order of the isometry.

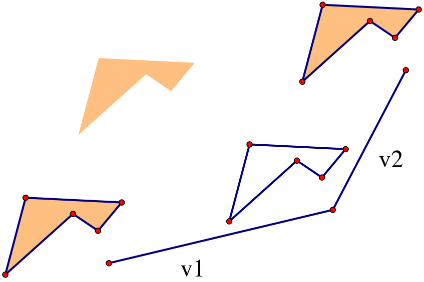
Draw as many conclusions as you can about the order of various isometries.

Conclusions about order

Isometry	Order
Translation in the direction of vector $\vec{v}$	
Rotation by angle θ in the counterclockwise direction around rotocenter point p	
Reflection across mirror line m	
Glide across mirror m and along vector $\vec{v}$ parallel to m	

Commutativity

Two isometries A and B **commute** if A followed by B gives the same result at B followed by A.



Experiment with commutativity

- Draw an irregular polygon and act on it with some isometry A. Then act on the image triangle with a second isometry B. Hide the middle image. Now fill the interior of the original triangle and act on the interior by B and then by A. Does the result exactly fill the image triangle you made by acting by A and then B? If so, you have evidence that the two isometries commute.
- Which isometries do you think commute and which do not? Note: it may be that, under some circumstances, certain types of isometries commute and under others they do not. Can you describe the circumstances in such cases?

Conclusions about commutativity

	Translation	Rotation	Reflection	Glide
Translation				
Rotation				
Reflection				
Glide				

Homework

1. What are the orders of the following isometries? a) rotation clockwise by 40 degrees, b) rotation counterclockwise by 135 degrees, c) rotation clockwise by 95 degrees? d) translation due north by 1 cm, e) reflection through a horizontal mirror line
2. Give an example of two isometries that do not commute.
3. For the symmetry group of the square, the identity (do nothing) isometry commutes with all the other isometries. Do any of the other 7 isometries commute with all the other isometries in the symmetry group of the square?