

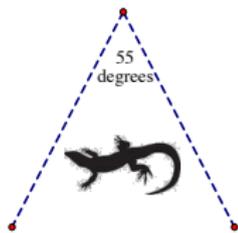
Part VII

Kaleidoscopes

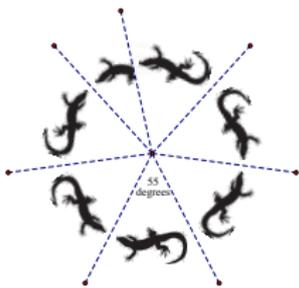
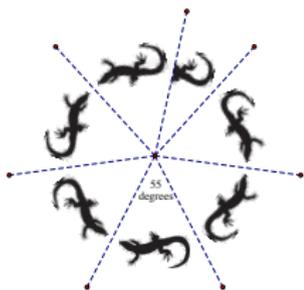
The goal for this lesson is to experiment with different mirror arrangements and the resulting symmetry patterns.

More with mirrors

You can make a hinged mirror out of two mirrors and tape, or use one of the hinged mirror kits. The mirrors should be upright and perpendicular to the table.

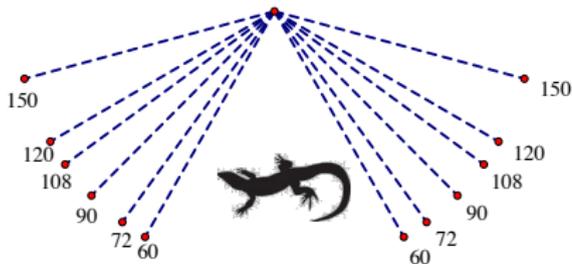


Place your hinged mirror on the lizard picture above, with the hinge at the dot and the mirrors lined up on the dotted lines. It is possible to see both of the images below. Describe how this can be done.



The number of lizards

Below is a lizard with lines forming various angles. For each angle, place your hinged mirrors so that the edges lie along the relevant dotted lines and the hinge is at the dot. Describe what you see.



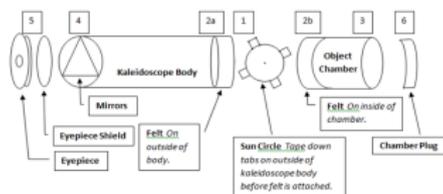
Angle measure measure	Number of images including the original	As many right-handed as left-handed lizards?
45°		
60°		
72°		
108°		
120°		
135°		
150°		
n°		

Give a general formula for the relationship between the angle and the number of lizards. When is there an equal number of lizards of each orientation?

Make a kaleidoscope

For this class, you will need cardboard tubes or PVC pipe of length 8 inches and diameter between 1.25 and 3 inches. Also bring scissors, tape, glue, and paper and markers or other materials for decorating the outside.

Kaleidoscope instructions



1. Fold down tabs of the transparent plastic "sun circle" and tape down onto end of kaleidoscope body. This piece will keep beads in the object chamber from falling into the main body of the kaleidoscope.
2. Attach 1/2 inch wide felt strips to the outside of one edge of the kaleidoscope body and the inside of one edge of the object chamber.
3. Slide the object chamber over the kaleidoscope body so that the felt strips line up and meet. The object chamber should extend past the body.
4. Form an equilateral triangle with three mirrors and push it into the kaleidoscope body. Remove the protective film before inserting the mirrors.
5. Drop the transparent eyepiece shield into the tube so that it lies flat against the mirrors.
6. Insert the eyepiece shield.
7. What do you think it will look like if you look through the kaleidoscope at an object in the room? Take a look and see if you were correct.
8. Add beads to the object chamber plug until about half of the plug is covered. Insert the plug into the object chamber.
9. Cut paper to fit around the kaleidoscope body and decorate.

Mutant kaleidoscopes

- ▶ The standard kaleidoscope has three mirrors that form an equilateral triangle.
- ▶ Experiment with using mirrors that form a triangle that is not equilateral, or a different polygon like a square.
- ▶ What configurations make whole numbers of shapes?

Animated kaleidoscopes

These instructions will first help you create a 2-mirror kaleidoscope with a finite image instead of the traditional 3-mirror kind, but can be adapted to create a 3-mirror kaleidoscope.

1. Use the Segment tool to draw a segments. Rotate the segment by 60 degrees around one of its vertices to make a "V".
2. Use the Polygon tool to draw a polygon inside the V. Or maybe two po
3. Double click on one of the segments of the V to mark it as a mirror line. Select both the polygon and the other segment of the V and choose Reflect from the Transform menu.
4. Continue reflecting polygon images AND segments across segments until you cannot get any new images. How many distinct polygons are there?
5. You can now experiment with different shapes of polygons by selecting and moving the vertices of the original polygon.
6. Select the polygon and choose Animate from the Display menu. Notice what is happening. The points defining the polygon are moving randomly on the plane. These points have been defined independently, meaning their locations were chosen independent of any other constructions.
7. You can gain some control over where the points can move by merging them with segments or circles (restricting their movements along those paths). To do so, click the Stop button on the Motion Controller (or choose Stop Animation from Display). Select one of the polygon vertices and then a segment of the V and choose Merge Point to Segment from the Edit menu. The point is now attached to the segment and can only move along that segment. (You also could have created a new segment to merge with the point.)
8. Use the Circle Tool in the toolbox to draw a circle in some blank space. Select another vertex of your polygon and then the circle and merge the vertex to the circle.
9. Hide the circles and any extra lines you have created, leaving just the polygons and mirror lines.
10. Again select the polygon interior and choose Animate.
11. You can use the Motion Controller to change speeds and directions of individual points.
12. Construct a 3-mirror kaleidoscope instead of a 2-mirror version, by drawing a third segment connecting the endpoints of the V to make an equilateral triangle. Then use this new segment as a mirror line and reflect the polygon across it. Continue reflecting the polygon and the segments and their images across mirror lines until you fill the canvas.
13. Animate your virtual kaleidoscope.

Homework

1. In class, we saw that hinged mirrors at an angle of $\theta = \frac{n}{360}$ will make n copies of the lizard, and if n is an odd number, then there will be a different number of left-tailed lizards than right-tailed lizards. Assuming that your original lizard is left-tailed, which odd numbers n will give you more left-tailed lizards than right-tailed lizards, and which will give you more right-tailed lizards than left-tailed lizards?
2. Draw a sketch or take a photo of your "kaleidoscope image": that is, what you see when you look through your kaleidoscope.
3. It's no surprise that your kaleidoscope image has reflection symmetry since it is made of mirrors.
 - 3.1 Does it also have (approximate) rotation symmetry? If so, through what rotation angle(s)?
 - 3.2 Does it also have (approximate) translation symmetry? If so, how long are the translation vectors and in what direction? (Your answer should be in terms of the length and position of the mirrors.)
 - 3.3 Does it also have (approximate) glide symmetry? If so, where are the glide lines and how long are the glide translation vectors?

(Remember, a "symmetry" is an isometry that leaves the figure unchanged. None of the symmetries for

our images will be exactly perfect, since we made the kaleidoscopes out of cardboard and plastic mirrors