Margin of Error

After completing this section, students should be able to:

• Explain the relationship between sample size and variability in sample results.
• Estimate a 95% confidence interval for a simple random sample given the sample percentage and the sample size.
Before class:

- Read the article Dem Caucuses could be a Five-Way Contest about the latest Monmouth University poll.
- (Optional) Read this description of How the Iowa Caucuses Work.
- Answer the questions on the before class quiz on Iowa caucuses.

1. What is the order of voters’ first choice candidates that the Monmouth poll predicts for the Iowa Caucus?
   (a) Biden, Sanders, Buttigieg, Warren, Klobuchar
   (b) Sanders, Biden, Warren, Buttigieg, Klobuchar
   (c) Sanders, Warren, Biden, Warren, Klobuchar, Buttigieg
   (d) Biden, Warren, Sanders, Buttigieg, Kobuchar

2. The Monmouth poll mentions two numbers of people: 544 and 1345.
   (a) 1345 is the population of likely Iowa caucus participants, and 544 is the size of the sample.
   (b) 1345 is the size of the sample selected and 544 is the number of people who responded.
(c) 1345 is the number of Iowa delegates sent to county conventions and 544 is the number of Iowa delegates sent to the national convention.

3. The article Dem Caucuses could be a Five-Way Contest quotes the Monmouth Polling Institute director as saying “Caucus electorates are the most difficult to model in polling.” Why might the outcome of the Iowa caucuses be difficult to predict from this poll? Select all reasons that might apply. (There are likely other reasons as well.)

(a) Voters’ second choice matters, not just their first choice, because if a candidate does not get 15%, they get no delegates, and their voters can re-sort and go to their second choice in the next — and final — round.

(b) Small shifts in who turns out to caucus can have a large impact on the outcome.

(c) There could be non-response bias, since only about 40% of the people contacted actually responded.

(d) Biden and Sanders are pretty close together at 23% and 21%.
Random samples are more informative than non-random samples, but there is still some variability of results.
Different polls give different results partly due to biases / inaccuracies but partly due to inherent variability, since you are never sampling the entire population, and each poll samples different people.
Goals

Today’s Goal: get a handle on how much variability there is in sample estimates
  • even when using a simple random sample
  • even with a 100% response rate
  • even in the absence of any kind of bias

Today’s Goal: make more sense of the Monmouth predictions for the Iowa caucuses and answer these questions
  • Is 23% for Biden vs. 21% for Sanders a big enough difference to trust?
  • What is meant by the margin of error of ±4.2 percentage points quoted in the article?
The *Monmouth University Poll* was conducted by telephone from January 23 to 27, 2020 with 544 Iowa voters who are likely to attend the Democratic presidential caucuses in February 2020, out of 1,345 registered Democrats and unaffiliated voters who were contacted for the poll. The question results in this release have a margin of error of +/- 4.2 percentage points. The poll was conducted by the Monmouth University Polling Institute in
Beads

In a real poll, we don’t have info about the real population of likely voters. But for today, we are going to use a population of beads that we do know about, to see how well our samples give us the right answers.

Suppose that the 4000 beads in the bin represent likely Democratic caucus goers from a specific region of Iowa, with the blue beads representing those who plan to vote for Joe Biden.

Note: the true proportion of blue beads is 0.295.
Overall question: If multiple samples of likely caucus goers were taken, how different will the estimates for the proportion of Biden voters (blue beads) be?
Task 1: Each student takes a sample of 4 beads with the spatula and counts the number of blue beads.

How will the proportion of blue beads (Biden voters) vary when collecting samples of FOUR beads (likely caucus-goers)?

What do you think the distribution of sample proportions will look like for samples of size 4? (PollEv)

Take a sample of size 4 and enter the proportion of blue beads in the PollEv poll.
Task 2: Each student takes a sample of TEN beads with the spatula and counts the number of blue beads.

What do you think the distribution of sample proportions will look like for samples of size 10? (PollEv)

Take a sample of size 10 and enter the proportion of blue beads in the PollEv poll.
Task 2: Each student takes a sample of TWENTY beads with the spatula and counts the number of blue beads.

What do you think the distribution of sample proportions will look like for samples of size 20? (PollEv)

(a) narrower than with samples of 10
(b) about the same as with samples of 10
(c) wider than with samples of 10

Take a sample of size 20 and enter the proportion of blue beads in the PollEv poll.
Observation: For larger sample sizes, you get (circle one) more / the same / less variability in the proportion, so (circle one) more / the same / less uncertainty in your estimates.

Conclusion: a larger sample size will give a (circle one) better / the same / worse estimate of the true proportion.
Margin of error and confidence intervals

Simulation of lots of samples from the bead box: population proportion is 0.295

About 95% of sample proportions are in the interval ____________.
About 95% of sample proportions are within __________ of the true proportion.

**Definition.** The 95% margin of error is the distance you have to go out from the center to capture 95% if the sampling distribution.

In this example, the 95% margin of error is __________

Note: you can also have a 90% margin of error or any other % margin of error, but 95% is most common.

If you hear *margin of error* without the % specified, assume it is 95%.
Simulation of lots of samples from the bead box: population proportion is 0.295

About 95% of sample proportions are in the interval __________.

About 95% of sample proportions are within __________ of the true proportion.
What is the margin of error?
Fill in the blanks. Recall that the margin of error here is 0.2.

For repeated samples of 20 (from a bead box with a proportion of 0.295 of blue beads), about 95% of the time, the ________________ will be within ± __________of the _________________. (PollEv)

A. sample proportion, 0.2, population proportion
B. population proportion, 0.2, sample proportion
C. it works either way

**Definition.** The 95% confidence interval is the interval sampling distribution ± margin of error

On the left side of the figure above, the circles represent sample proportions and the horizontal lines represent 95% confidence intervals.
Fill in the blanks and answer the True False questions. Recall that the margin of error here is 0.04.

True or False: About 95% of the time, the sample proportion will be within 0.04 of the true proportion.

True or False: About 95% of the time, the true proportion will be within 0.04 of the sample proportion.
True or False: About 95% of the time the sample proportion will be in the interval given by
population proportion ± margin of error

True or False: About 95% of the time the population proportion will be in the interval given by
sample proportion ± margin of error

Which of the previous two statements are more useful in practice?

On the right side of the figure, the 95% confidence intervals are shown for each of 100 samples. For how many samples does the 95% confidence interval contain the true population proportion? For how many samples does it NOT? Is this about a 95% / 5% split?
Margin of error and confidence intervals

StatKey: Sampling Distribution for a Proportion

Sampling Dotplot of Proportion

Margin of error and confidence intervals

MARGIN OF ERROR

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In the previous examples, the margin of error was computed using the true sample proportion. But normally, we don’t know the true population proportion. We ONLY know the proportion for ONE sample.

For example, in the Monmouth University poll, we only know that in the sample of 544 people, 0.23% favor Biden.

So we use the SAMPLE population instead as a proxy for the TRUE population proportion to estimate the margin of error.
Simulation of lots of samples of 544 for Iowa poll: suppose population proportion is really 0.23

What is our estimate for the margin of error?
Formulas

The margin of error can be computed by simulation as above (computer app like StatKey or repeatedly physically re-sampling beads from our sample, with replacement) This is called “bootstrapping”.

OR

The margin of error can be computed by a formula $2 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$ where $\hat{p}$ is the sample proportion and $n$ is the sample size.
Back to the polls

Back to the Monmouth University Iowa poll, 544 people were interviewed and 23% favored Biden ...

Compute the margin of error using the formula $2 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$

Does this agree with the bootstrap estimate for the margin of error that we got from Statkey?

What is a 95% confidence interval for Biden’s percent of the vote?

What do you think of Biden’s 23% to 21% lead over Sanders in this poll? (PollEv)
A. Based on this poll, we can be pretty confident Biden will win.
B. Based on this poll, it’s too close to call.
But the Monmouth poll article gives a “maximum” margin of error of \( \pm 4.2 \), which is slightly bigger than the margin of error based on the StatKey graph (and from the formula  
\[
2 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}
\]
for the sample of likely Democratic caucusgoers, one can say with 95% confidence that the error attributable to sampling has a maximum margin of plus or minus 4.2 percentage points.

Where did this “maximum margin” come from?
A Jan 29th Iowa University poll puts Bernie Sanders in the lead:

Of the 655 likely Democratic caucus-goers surveyed, 24% said Sanders was their first choice. Sen. Warren was second with 19%, followed by Mayor Pete Buttigieg with 17% and Joe Biden with 15%. Peterson said Sen. Amy Klobuchar remains in fifth, but more than doubled her support from last month at 11%.

The survey has a margin of error of plus or minus 4.8%.

What is the 95% confidence interval for Biden’s percentage of votes?

Can the discrepancy between this poll and the Monmouth poll be accounted for by chance variability? (PollEv)

A. Yes, a difference like that is to be expected due to chance.
B. No, there is likely something else going on that makes the results different.
1. Lots of samples were taken from a box of beads with 40% red beads. For each sample, the sample’s proportion of red beads was recorded. The distributions of sample proportions were graphed below. In one of the graphs, lots of samples were taken with sample size 50, in another, the samples all had sample size 240, and in a third, the samples had sample size 600.

(a) Which graph is which?
(b) How can you tell, just by comparing the shapes of the graphs?

2. (6 pts) The following graph shows a sampling distribution for sample proportions of green beads for samples of size 150 from a population which is 56% green beads.
(a) Circle or shade in the part of the distribution that represents the roughly 95% of sample proportions that are closest to the true population proportion.

(b) Fill in the blank: About 95% of sample proportions are within plus or minus ______ of the true population proportion of 0.56.

(c) Supposed you take one sample of 150 beads from this population and get 80 green beads. What would be your estimate of the population proportion of green beads, based on this one sample?

(d) Approximately what would be your margin of error for this one sample from part (c)?

(e) Approximately what would be your 95% confidence interval for this one sample from part (c)?

(f) Does your 95% confidence interval from part (e) include the true population proportion?
3. A Jan 26 Suffolk University / USA Today poll says

“Biden was backed by 25% of likely Democratic caucusgoers, ahead of Vermont Sen. Bernie Sanders at 19%; former South Bend, Indiana, Mayor Pete Buttigieg at 18%; Massachusetts Sen. Elizabeth Warren at 13%; and Minnesota Sen. Amy Klobuchar at 6%. The survey was of 500 likely voters and has a margin of error of plus or minus 4.4%.”

(a) What is the 95% confidence interval for Biden’s percentage of votes in this USA Today poll?

(b) Do you think the discrepancy between this poll’s numbers for Biden (25%) and the Monmouth poll’s number for Biden (23%) could be accounted for by chance variability? Why or why not? Mention margin of error in your explanation.

(c) The USA Today article about this poll goes on to say:

““The Suffolk/USA TODAY Poll, taken Jan. 23-26, is the most current in a string of recent Iowa polls. A New York Times/Siena poll taken Jan. 20-23 showed Sanders leading at 25%, Buttigieg at 18% and Biden at 17% .... The differing results could reflect changes in public sentiment or simply the effects of the margins of errors in polls.”

Do you agree that the discrepancy between Biden’s percents in the USA Today poll (25%) and the NYTimes poll (17%) could simply reflect the effects of mar-
gins of errors [i.e. variability] in polls? Why or why not? Mention margin of error in your explanation. In case you are wondering, NYTimes poll sampled 584 voters and the margin of error was plus or minus 4.8 percentage points.