After completing this section, students should be able to:

- Explain where the formula for compound interest comes from.
- Solve for the final amount of money from the initial amount of money, or vice versa, given information about the interest rate, the compounding period, and the number of years.
- Solve for the final amount of money from the initial amount of money, or vice versa, for continuously compounded interest, given information about the interest rate and the number of years.
- Compare the money accumulated when interest is compounded at different time periods and continuously.
- Assess present and future value of an investment
Before Class

• Watch the 11 minute video on compound interest

• Answer the questions

(a) You invest $200 in a savings account that earns 4% annual interest, compounded once a year. If you make no deposits or withdrawals, how much will you have in the account after 3 years?

(b) Suppose you invest $300 in an account that earns 5% annual interest, compounded monthly. After 6 years, how much will you have?

(c) You invest $700 in a bank account with a 2.5% annual interest rate (APR), compounded continuously. After 5 years, how much will you have?
Introduction

**Example.** Match the equations with the situations: (PollEv)

1. \( y = Pe^{rt} \)
   - A. Money earns 5% interest once a year.

2. \( y = P(1 + r)^t \)
   - B. Money earns 2% interest compounded monthly.

3. \( y = P\left(1 + \frac{r}{n}\right)^{nt} \)
   - C. Money earns interest compounded continuously, at an APR of 4%.

What do the variables mean in these equations?
Example. Suppose you invest $4000 at a 8% interest rate compounded monthly. What percent interest do you actually earn on your money after one year? (PollEv)

A. Less than 8%
B. Exactly 8%
C. More than 8%

Definition. The Annual Percentage Rate (APR) is ...

Definition. The Annual Percentage Yield (APY) is ...
Question. If you invest $1000, how much money do you have after a year,
(a) at a 5% APR compounding annually?
(b) at a 5% APR compounded monthly?
(c) at a 5% APR compounded daily?
(d) at a 5% compounded continuously?

Question. What is the APY (annual percentage yield) in each of these cases?
Present value

Example. The next total solar eclipse visible in the US will be on April 18, 2024. You want to have $800 available by then to go on a road trip to see it. How much would you need to put aside now, with 3% annual interest compounded monthly, to have $800 available by then? (PollEv)

A. $662
B. $710
C. $747
D. $792

The number you calculated is called the present value of $800.

$800 is called the future value of ...

Definition. The present value of an amount of money \( M \) after \( t \) years is ...
Extra Example. Your grandparents are loaning you $5000 to buy a car. Instead of making monthly payments, they ask that you pay back all the money in 10 years, with interest, all as a lump sum. They give you three options:

Option 1: Annual interest rate of 9.0% compounded once a year.

Option 2: Annual interest rate of 8.95%, compounded monthly.

Option 3: Annual interest rate of 8.95%, compounded continuously.

Which is best for you? Which is worst?
Example. You just received a stimulus bonus for $1,200, and you’re going to deposit it in an account that earns 4% interest, compounded continuously.

(a) How much will you have after 5 years?

(b) You’d like to buy a computer that costs $1500. How long will you have to wait before your money has earned that much? (Assume the computer price doesn’t change.)
Homework

(a) Suppose a person invests their $1200 of stimulus money at 3% APR compounded continuously.
   i. What is the APY?
   ii. How much money will they have in 3 years?
   iii. How long will it be until they have $1500?

(b) You have just won a $1 million lottery. This new lottery, however, will pay out the award 50 years from today. What is the present value of your award based on a 8% annual interest rate, compounded monthly?