

Compound Interest

After completing this section, students should be able to:

- Explain where the formula for compound interest comes from.
- Solve for the final amount of money from the initial amount of money, or vice versa, given information about the interest rate, the compounding period, and the number of years.
- Solve for the final amount of money from the initial amount of money, or vice versa, for continuously compounded interest, given information about the interest rate and the number of years.
- Compare the money accumulated when interest is compounded at different time periods and continuously.
- Assess present and future value of an investment

Introduction

Example. Match the equations with the situations: (PollEv)

1. $y = Pe^{rt}$

A. Money earns 5% interest once a year.

2. $y = P(1 + r)^t$

B. Money earns 2% interest compounded monthly.

3. $y = P\left(1 + \frac{r}{n}\right)^{nt}$

C. Money earns interest compounded continuously, at an APR of 4%.

What do the variables mean in these equations?

APR and APY

Example. Suppose you invest \$4000 at a 8% interest rate compounded monthly. What percent interest do you actually earn on your money after one year? (PollEv)

- A. Less than 8%
- B. Exactly 8%
- C. More than 8%

Definition. The Annual Percentage Rate (APR) is ...

Definition. The Annual Percentage Yield (APY) is ...

Question. If you invest \$1000, how much money do you have after a year,

- (a) at a 5% APR compounding annually?
- (b) at a 5% APR compounded monthly?
- (c) at a 5% APR compounded daily?
- (d) at a 5% compounded continuously?

Question. What is the APY (annual percentage yield) in each of these cases?

Present value

Example. . The next total solar eclipse visible in the US will be on April 18, 2024. You want to have \$800 available by then to go on a road trip to see it. How much would you need to put aside now, with 3% annual interest compounded monthly, to have \$800 available by then? (PollEv) Choose the closest answer.

- A. \$701
- B. \$729
- C. \$733
- D. \$777
- E. \$794

The number you calculated is called the *present value* of \$800.

\$800 is called the *future value* of ...

Definition. The *present value* of an amount of money M after t years is ...

How long will it take?

Example. You just received a stimulus check for \$1,400, and you're going to deposit it in an account that earns 4% interest, compounded continuously.

You'd like to buy a computer that costs \$1750. How long will you have to wait before your money has earned that much? (Assume the computer price doesn't change.)

Hint: you can figure this out by solving an equation for t using logs, but you can also figure this out with a spreadsheet by calculating a list of amounts corresponding to a list of values for t .

Simple Loans

Extra Example. Your grandparents are loaning you \$5000 to buy a car. Instead of making monthly payments, they ask that you pay back all the money in 10 years, with interest, all as a lump sum. They give you three options:

Option 1: Annual interest rate of 9.0% compounded once a year.

Option 2: Annual interest rate of 8.95%, compounded monthly.

Option 3: Annual interest rate of 8.95%, compounded continuously.

Which is best for you?