Review Sheet on Financial Topics

This review sheet covers:

- Expected value
- Estimating health care costs and frequencies of use
- Compound interest

Expected Value

1. The expected value of a game is the amount of money or points you would expect to win, on average, if you played the game over and over again.

2. If the expected value is positive, you would expect to win money on average. If it is negative, you would expect to lose money on average. If it is zero, you would expect to come out even on average.

3. You can calculate the expected value by charting the outcomes and their probabilities, and then adding up the probability times the outcome. For example, suppose if you are taking a test and guessing on a bunch of multiple choice questions with 5 answer choices each, and you earn 2 points for the right answer and lose a quarter point for each wrong answer. Then the expected value is \( \frac{1}{5} \cdot 2 + \frac{4}{5} \cdot \left( -\frac{1}{4} \right) = \frac{1}{5} \), so
you would expect to earn about $\frac{1}{5}$ of a point on average by guessing.

Health Insurance

4. To figure out which health insurance plan is best, you need to estimate the cost of procedures, their probability, and what fraction of the cost or dollar amount you would pay with the insurance. You also need to add in the cost of the insurance premiums and consider the deductible.

5. For example, you can make estimates of the probability of a doctor visit in a year by finding the total number of visits in the US in a year and dividing by the population.

6. For example, you can make estimates for the probability of getting a filling during a year by taking the average total number of fillings in adult teeth that people of a certain age have and dividing by the number of years they had adult teeth (their age minus about age 6).

7. In most situations, it is cheaper to get no insurance than insurance, since the insurance company has to be making a profit. This might not be true if the insurance is subsidized by your employer, or you have higher than average health costs. You may want to get insurance anyway to avoid having to pay for low
probability high cost events that could bankrupt you.

**Compound Interest**

8. For interest compounded once a year: \( y = P(1 + r)^t \), where \( P \) is the initial deposit, \( r \) is the interest rate as a decimal (e.g. 5% annual interest is \( r = 0.05 \)), \( t \) is time in years, and \( y \) is the final amount.

9. For interest compounded \( n \) times a year: \( y = P (1 + \frac{r}{n})^{nt} \), where \( P \) is the initial deposit, \( r \) is the interest rate as a decimal (e.g. 5% annual interest is \( r = 0.05 \)), \( t \) is time in years, \( y \) is the final amount, \( n \) is the number of compounding periods a year (e.g. daily compounding means \( n = 365 \)).

10. For interest compounded continuously: \( y = Pe^{rt} \), where \( P \) is the initial deposit, \( r \) is the interest rate as a decimal (e.g. 5% annual interest is \( r = 0.05 \)), \( t \) is time in years, and \( e \) is the constant \( e = 2.718 \ldots \). 

11. The amount of money you get compounding continuously at interest rate \( r \) is the limit of the amount of money you get compounding at shorter and shorter time intervals: monthly, weekly, daily, hourly, every second, etc.

12. The APR (annual percentage rate) is the advertised annual rate. The APY (annual percentage yield) is the actual percent interest that the money earns after a year.
The APY is equal to the APR if the interest is only compounded once at the end of the year; otherwise the APY is larger because you are earning interest on interest.

13. The future value of an amount of money (say $100) is the amount of money you will get from investing that amount of money later, after a specified number of years at a specified interest rate and time of compounding. For example, the future value of $100 invested at an APR of 7% compounded continuously for 10 years is $100 \cdot e^{0.07 \cdot 10} = 201.38$.

14. The present value of an amount of money (say $100) is the amount of money you would have to invest now in order to get that amount of money after the specified time at the specified interest rate and style of compounding. For example the present value for $100 invested at an APR of 7% compounded continuously for 10 years can be found by setting $100 = P \cdot e^{0.07 \cdot 10}$ and solving for $P$ to get $P = 49.66$.

15. To find out how long it will take your investment to reach a certain value, you can use logs. For example, to find out how long it will take $500$ to grow to $700$, if you invest it at a 7% interest rate, compounded monthly, solve $700 = 500 \cdot (1 + \frac{0.07}{12})^{12t}$ for $t$ by dividing by 500, then taking the log of both sides:

$$\frac{700}{500} = (1 + \frac{0.07}{12})^{12t}$$
\[
\ln\left(\frac{700}{500}\right) = \ln\left((1 + \frac{0.07}{12})^{12t}\right)
\]

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\ln\left(\frac{700}{500}\right) = 12t \ln\left(1 + \frac{0.07}{12}\right)
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t = \frac{\ln\left(\frac{700}{500}\right)}{12 \ln\left(1 + \frac{0.07}{12}\right)} = 4.82 \text{ years}
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