SIR Disease Model Introduction

After completing this section, students should be able to:

- Explain the structure of a Markov model and what information is needed to build one
- Use a spreadsheet with formulas to build an SIR model, change assumptions, and draw conclusions

Introduction

Suppose we wanted to describe the behavior of a large group of puppies. We might define states of activity (called state variables), like

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And we could give probabilities for transitioning between states each minute.

Over time, we could simulate what is happening to the puppy population. This sort of model is called a Markov model.

For a disease, like coronavirus, we could also establish states. What might we use for our state variables?

For now, let's make up some probabilities for transitioning between states, each day.

Transition game

1. The three sections of the room will correspond to the disease states. Everyone start in the Susceptible section, like we all were susceptible at the start of the covid pandemic.
2. Find a random number generator (e.g. google "Random number generator") and set it to pick a random number between 1 and 10.
3. When the instructor says it's time to transition, generate a random number on your phone and use it to transition between states using the following rules:

What happens in the long run?

Why is this enactment not a realistic model of an actual disease progression? How could we make it more accurate?

Variables and equations

Define some variables:

- Let t represent
- Let $S(t)$ represent
- Let $I(t)$ represent
- Let $R(t)$ represent
- We can split $R(t)$ into two groups:
- Let $N(t)$ represent

How is $S(t+1)$ related to $S(t)$ and the number of newly infected people?

$S(t+1) = S(t) \pm$ __________________

How is $I(t+1)$ related to $I(t)$ and the number of newly infected people and the number of newly removed people?

$I(t+1) = I(t) \pm$ __________________

How is $R(t+1)$ related to $R(t)$ and the number of newly infected people and the number of newly removed people? (PollEv)

A. $R(t+1) = R(t) -$ number of newly infected people + number of newly recovered people

B. $R(t+1) = R(t) +$ number of newly infected people − number of newly recovered people

C. $R(t+1) = R(t) +$ number of newly infected people

D. $R(t+1) = R(t) +$ number of newly removed people

E. $R(t+1) = R(t) -$ number of newly removed people

Next, let's try to describe the number of newly infected people in more detail.

Do you expect the number of newly infected people each day to be bigger or smaller or the same if $I(t)$ is bigger?

Do you expect the number of newly infected people each day to be bigger or smaller or the same if the the fraction of the population that is susceptible (and not immune) is bigger?

What if the number of susceptible people gets bigger, assuming the population size stays constant?

For now, we will take on faith that the number of newly infected people each day is proportional to ...

which is plausible because ...

Recall that

$S(t+1) = S(t) -$ ____________________

This gives us the equation

$S(t+1) = S(t) -$ ____________________

where b is a constant of proportionality.

Next, let's try to describe the number of newly removed people in more detail.

Do you expect the number of newly removed people each day to be bigger or smaller or the same if $I(t)$ is bigger?

Do you expect the number of newly removed people each day to be bigger or smaller or the same if $S(t)$ is bigger?

For now, we will take on faith that the number of newly removed people each day is proportional to ...

which is plausible because ...

Recall that

$R(t + 1) = R(t) +$ ____________________

This gives us the equation

$R(t + 1) = R(t) + ...$

where a is a constant of proportionality.

Recall that

$I(t + 1) = I(t) +$ ____________________ $-$ ____________________

Therefore,

$I(t + 1) = I(t) +$ ____________________ $-$ ____________________

where a and b are constants of proportionality.

Initial conditions means conditions at time $t = 0$.

What initial conditions do we need to set?

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What other parameters do we need to set?

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Let's put this all into a spreadsheet. Open up the Google spreadsheet linked to on Canvas

Set the parameters to $a = 0.1$ and $b = 0.00005$ and the initial conditions to $S(0) = 20,000$, $I(0) = 100$, and $R(0) = 0$.

Use the spreadsheet and your brain to answer the questions on this online worksheet from mathinsight.org.

a. Given these initial conditions, how many people were sick on day zero?

   How many were susceptible on day zero?

   How many people were susceptible on day zero and then became sick on the first day?

   How many people were sick on day zero and then recovered on the first day?

b. After the first day, what is the total number of people who are susceptible?

   What is the total number of infectives?

How many have recovered from the disease?

c. Determine the values of the three state variables (S(t), I(t), and R(t)) after 30 days and after 60 days

What happens to the number of susceptible people as time passes? Therefore, how severe was the disease?

d. What does the parameter b mean? It controls the rate that (circle one) susceptible / infective / removed . people (circle one) get sick / recover or die.

Therefore, if we decrease b, what should happen to the course of the disease? The outcome of the disease should be (circle one) more / less severe.

e. What does the parameter a mean? It controls the rate that (circle one) susceptible / infective / removed people (circle one) get sick / recover or die.

Therefore, if we increase a, what should happen to the course of the disease? The outcome of the disease should be (circle one) more / less severe.