The SIR Disease Model Parameters

After completing this section, students should be able to:

- Explain the difference between basic reproductive number and effective reproductive number
- Use data on effective reproductive number and length of infectious period to fill in parameters for an SIR model

Reproductive numbers

Epidemiologists usually describe a disease in terms of the BASIC reproductive number r_0, the EFFECTIVE reproductive number r_e, and the average length of the infectious period k.

r_0, the basic reproductive number of a disease, is ...

r_e, the effective reproductive number, is ...

It can also be described as the number of infecting contacts each infected person makes on average if ...

r_e is generally (circle one) lower / higher than r_0 because ...

Can r_0 change over time? If so, would you expect it to go up or down?

Can r_e change over time? If so, would you expect it to go up or down?

Supposed $r_0 = 4.5$, but only 2/3 of the population is susceptible, and 1/3 is immune. What will r_e be?

If $S(t)$ is the number of susceptible people on day t, $I(t)$ is the number of infectious people on day t, $R(t)$ is the number of removed people at time t, and $N(t)$ is the number of people at time t, relate r_e to r_0 and some of those other quantities. Hint: think about the previous problem.

Unpacking b

Suppose r_0 is 4.5, and the disease is contagious, on average, for 6 days. How many infecting interactions would each infected person make, on average, PER DAY, in a fully susceptible population?

If r_0 is the basic reproductive number and k is the average length of time a person is infectious, write an expression for the average number of infecting interactions that each infected person makes per day, if everyone is susceptible.

A. $r_0 \cdot k$

B. $\frac{r_0}{k}$

C. $\frac{k}{r_0}$

D. $\frac{1}{k \cdot r_0}$

Is it reasonable that a person makes the same number of contacts each day?

Now suppose that not everyone is susceptible. Some are immune. Write a formula for the average number of infecting contacts that an infected person makes per day.

Write a formula for the number of new infections per day. Hint: think about the previous question and answer.

Now we are ready to rewrite the formula $S(t+1) = S(t) -$ number of newly infected people that day. Write this formula with the number of newly infected people filled in with your previous answer.

Recall that earlier, we took on faith that $S(t+1) = S(t) - bS(t)I(t)$.

- Does this agree with the analysis we just did?

- Write b in terms of r_0 and other quantities. Assume $N(t)$ is constant. (PollEv)

  A. $b = \dfrac{r_0}{k}$

  B. $b = \dfrac{r_0}{k \cdot N}$

  C. $b = \dfrac{1}{k \cdot r_0}$

  D. $b = \dfrac{1}{k \cdot r_0 \cdot N}$

  E. $b = \dfrac{k \cdot r_0}{N}$

Unpacking *a*

In the formula $R(t+1) = R(t) + aI(t)$, what does a represent? (PollEv)

A. The fraction of the infected people who are removed every day.

B. The number of infected people who are removed every day.

C. The fraction of the infected people who stay sick every day.

D. The number of infected people who stay sick every day.

E. The time it takes for an infected person to recover.

If every day, about 1/10 of the people who are sick recover, then some people will recover very quickly and some people will recover very slowly, but it turns out on average, the recovery time is 10 days.

Similarly, if every day, about 1/k of the people who are sick recover, then on average, recovery time will be _____ days.

So, if we know recovery time is k days, then the parameter a should be ...

Parameters for covid

What is the average contagious period for covid-19? What should be set a to?

- As of Feb 21, 2021, the CDC suggests people are no longer contagious after 10 days, possibly 6 days after developing symptoms. See Interim Guidance on Duration of Isolation and Precautions for Adults with COVID-19
- People may be infectious 48 hours before developing symptoms. Source: Harvard Health Publishing

What is r_0 for coronavirus?

What should we use for b for coronavirus?

SIR Spreadsheet

Put reasonable parameters for a and b into the spreadsheet model and see what happens. Predict the course of the infection in the US.

Observations?

Effective reproductive number

What is the relationship needed between a and b so that the number of infectives will go down, not up?

Hint: $I(t+1) = I(t) + bS(t)I(t) - aI(t)$

Translate this inequality into an inequality involving r_e.

How small would r_0 have to be for an epidemic to die out immediately, without ever getting bigger, in a population in which everyone is susceptible?

What measures can we take to decrease r_0?

On the SIR model, verify that the daily number of new cases starts going down when $r_e < 1$, and that when $r_0 < 1$, the epidemic never takes off in the first place.

Critique assumptions

"All models are wrong. Some are useful." -George Box

What are some of the simplifying assumptions that we made for the SIR model?