Covid Vaccination

After completing this section, students should be able to:

- Define herd immunity
- Explain why an epidemic will start to die down when $r_e < 1$.
- Explain the relationship between the basic reproductive number of a disease and the level of vaccination needed to squash an outbreak.
- Use this relationship to calculate levels of vaccination needed.

Let's see what happens if we add vaccination to our SIR model. Vaccinating people in advance of a disease outbreak affects which of these initial conditions? (Choose the best answer(s)) (PollEv)

A. $S(0)$

B. $I(0)$

C. $R(0)$

Which other parameters in our SIR model will vaccinating affect?

A. r_0

B. r_e

C. a

D. b

Vaccination in the SIR model

Return to the SIR the spreadsheet, and using a contagious period $k = 10$ days, $r_0 = 6.5$, $S(0) = 100000$, $I(0) = 100$, $R(0) = 0$, and the population size $N(0) = 100100$.

Make a cell that gives the vaccination level and set it at 0.5.

Now update $R(0)$ to account for this level of vaccination, without changing the population size, by using a formula. Using a formula means we can tweak this vaccination level easily and see the effects.

Update $S(0)$ also using a formula.

Does $I(0)$ need to be updated to keep the population size the same and keep the model making sense?

What do you notice from the graph about the epidemic?

Experiment with different levels of vaccination at time 0 by changing the vaccination rate, and $S(0)$ and $R(0)$ accordingly. What is the lowest level of vaccination that makes the number of infected people go strictly down, without ever going up?

Does this agree with the formula $V = 1 - 1/r_0$ from the video?

Try using a different value of r_0, and again, experiment with the level of vaccination until you have the smallest vaccination rate that makes the number of infected people go down, without going up. Check that it again agrees with the $1 - 1/r_0$ formula.

The formula $1 - 1/r_0$

Let's see if we can explain where the vaccination level formula $1 - 1/r_0$ comes from.

Why should an epidemic die down if $r_e < 1$?

Write down a formula relating r_0 and r_e.

Assume you vaccinate at time 0, and no one is already immune from prior infections (e.g. a new disease)

The vaccination level V (the fraction of the population that is vaccinated) is related to

- $S(0)$ (the number of susceptible people at time 0) and
- $N(0)$ (the population size at time 0)

in the following way:

A. $\dfrac{S(0)}{N} = V$

B. $\dfrac{S(0)}{N} = 1 - V$

C. $\dfrac{S(0)}{N} = 1 + V$

D. $\dfrac{S(0)}{N} = V - 1$

Recall that $r_e = r_0 \cdot \dfrac{S(t)}{N(t)}$. So at time 0, $r_e = r_0 \cdot \dfrac{S(0)}{N(0)}$.

Use the previous two equations to rewrite the inequality $r_e < 1$ in terms of V.

Solve for V to get a formula that looks familiar

Definition. Herd immunity is ...

Use the formula $V = 1 - 1/r_0$ to see what vaccination level would be needed to establish herd immunity for a disease with $r_0 = 4.5$, in a population that has never been exposed to the disease.

Suppose we can only achieve a vaccination level of 55% for this disease, but can reduce r_0 by social distancing and other measures. What level would we have to reduce r_0 to, in order to achieve herd immunity?

Critique assumptions

”All models are wrong. Some are useful.” -George Box

What are some of the simplifying assumptions that we made for the SIR model?