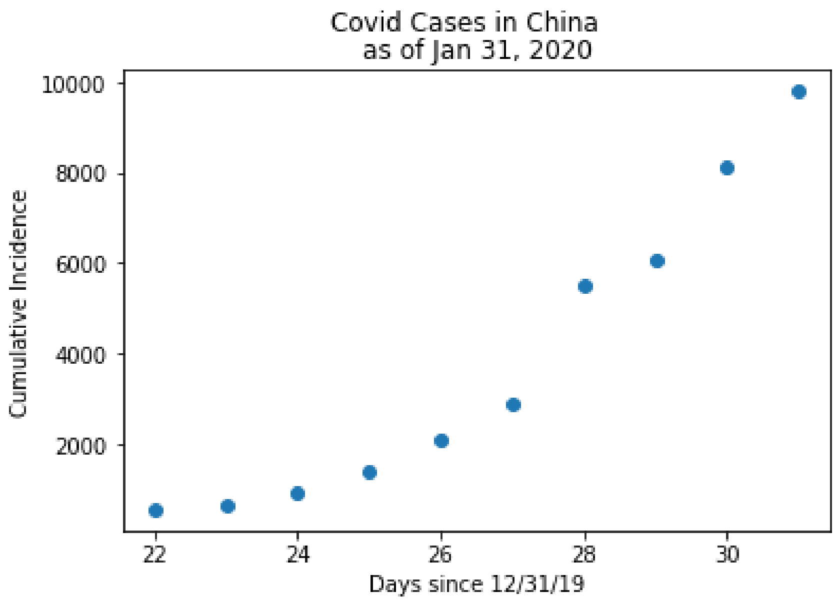
Covid and Exponential Growth

After completing this section, students should be able to:

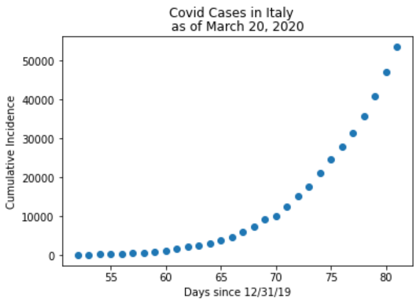
- Distinguish between exponential vs. linear growth based on graphs
- Explain why taking the log of the y-values of a data set can transform an exponential relationship between x and y to a linear one.
- Make predictions about the spread of coronavirus based on exponential growth, and evaluate the weaknesses of those predictions.

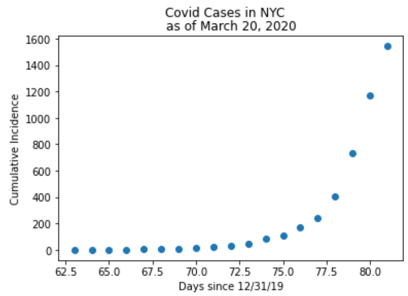
The covid-19 pandemic started in China. Early on, the cumulative number of cases (cumulative incidence) looked like this. The numbers on the x-axis represent days since Dec 31, 2019, and the y-axis represents cumulative incidence.

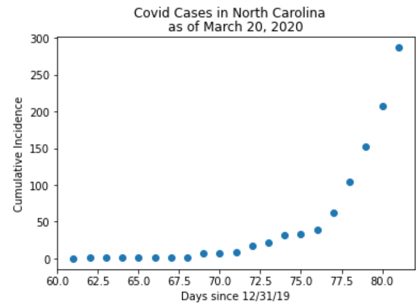


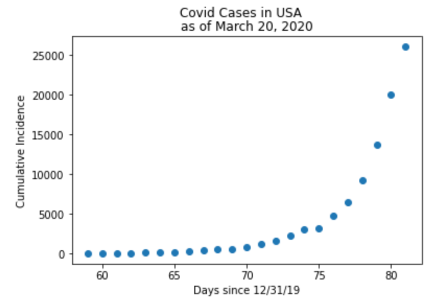
What is meant by cumulative incidence?

This same pattern was repeated in other locations.





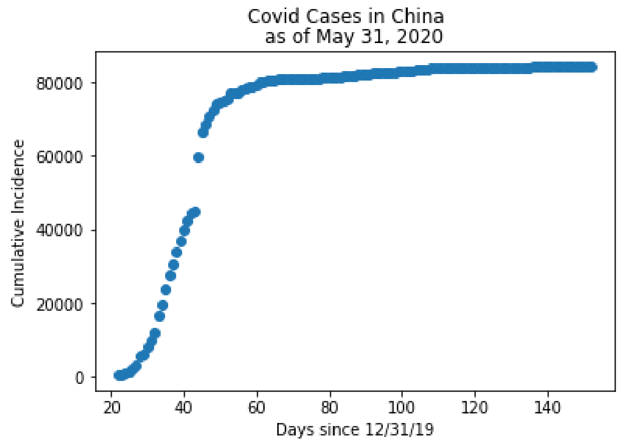




How would you describe the shape of these curves?

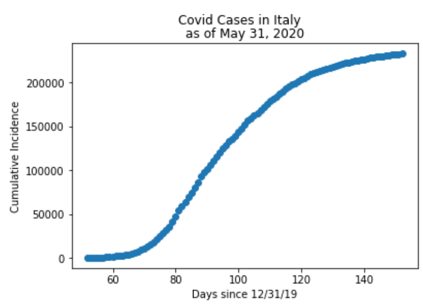
Why might it be useful to fit curves (i.e. equations $y = f(x)$) to the data on these plots?

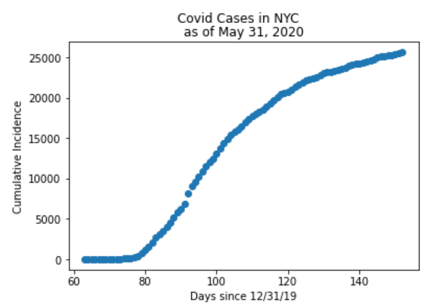
Later on in the pandemic, the plots of cumulative number of cases in China looked like this.

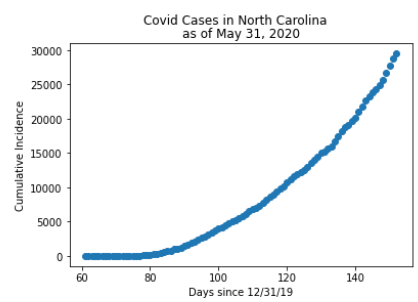


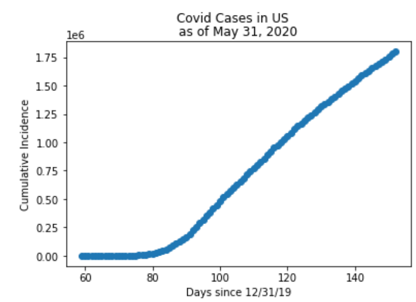
What is the name of this type of curve?

Here are some plots from May 31, 2020 in other locations.







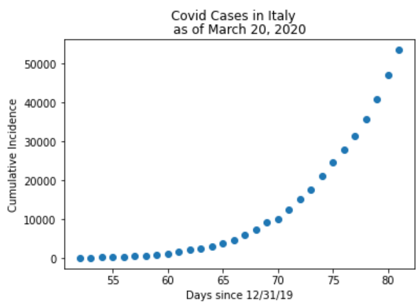


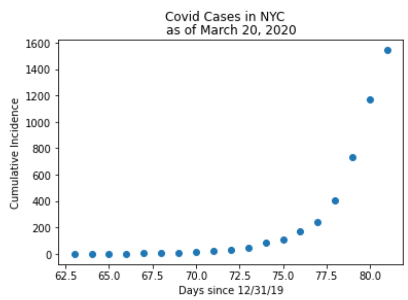
Why might it be useful to fit curves (equations) to the data on these plots?

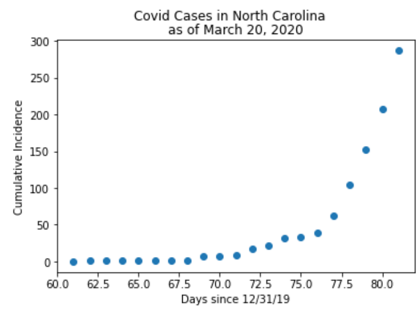
Today we will focus on fitting exponential functions to early covid data. Later on we will fit logistic curves.

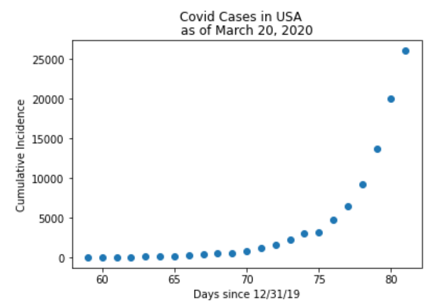
An exponential function is a function of the form

On March 20, 2020, which of these locations appear to be experiencing exponential growth of cumulative incidence of covid? Select all that apply.









A. Italy

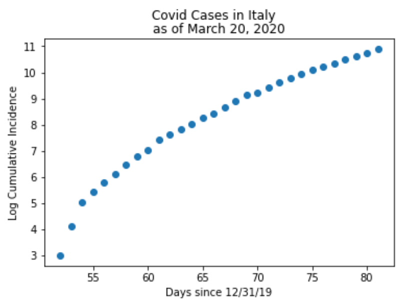
B. NYC

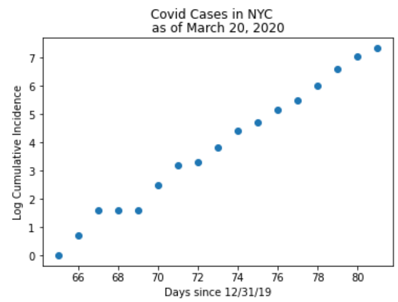
C. NC

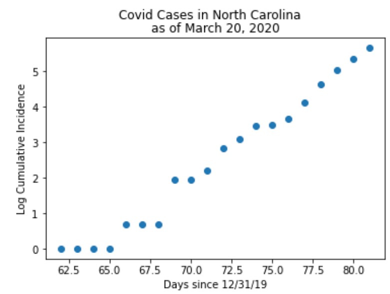
D. US

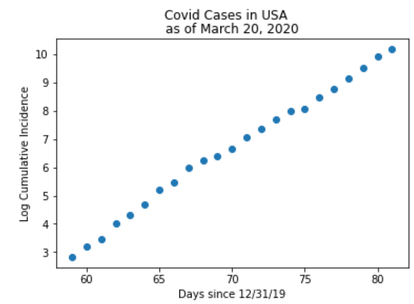
It can be difficult to distinguish exponential growth from non-exponential growth just from looking at a plot.

It can be easier to see what is going on by looking at a *semi-log plot*, in which we plot the log of the y-variable against the x-variable.





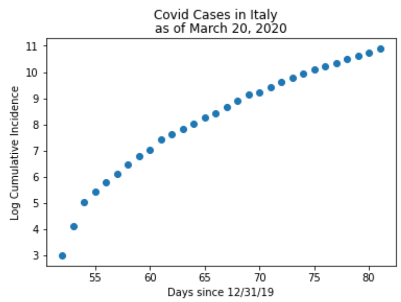


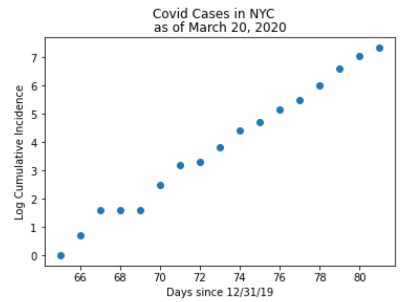


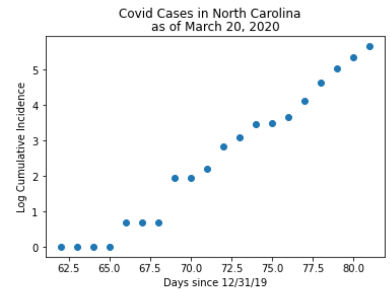
What do you notice about the shapes of these plots?

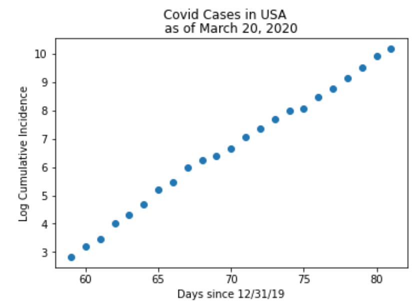
Fact: If relationship between two variables x and y is given by the exponential function $y = ae^{kx}$, then the relationship between x and $\ln(y)$ is ...

On March 20, 2020, which of these locations appear to be experiencing exponential growth of cumulative incidence of covid? Select all that apply.









A. Italy

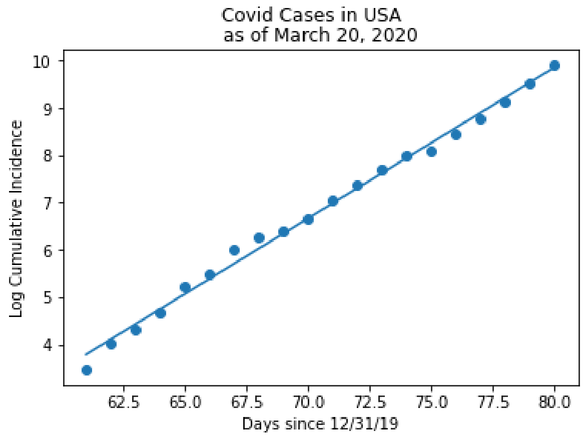
B. NYC

C. NC

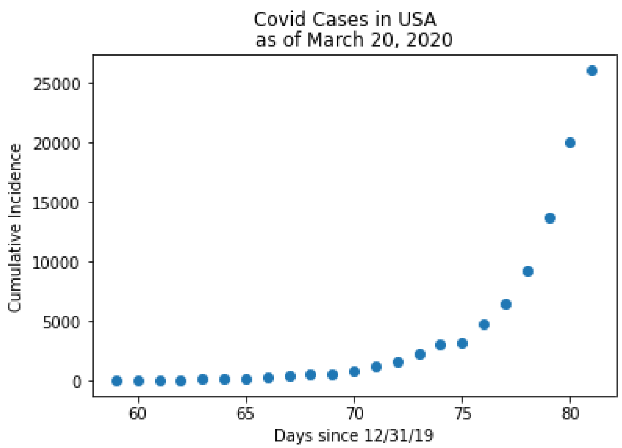
D. US

USA

It turns out that the line $y = mx + b$ with $m = 0.32$ and $b = -15.73$ is a good fit to the US semi-log plot of cumulative incidence over time.



What would be the best equation to fit to the original plot?



Based on this equation, when do you think the first US case happened?

What is the growth factor, i.e. the amount by which cumulative covid cases is multiplied by each day?

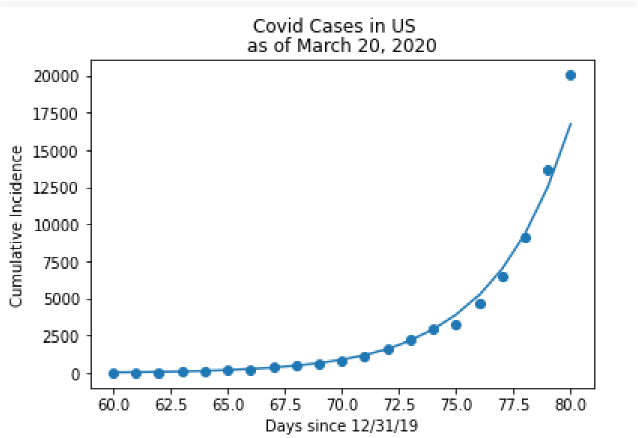
What was the percent increase in cumulative covid cases per day?

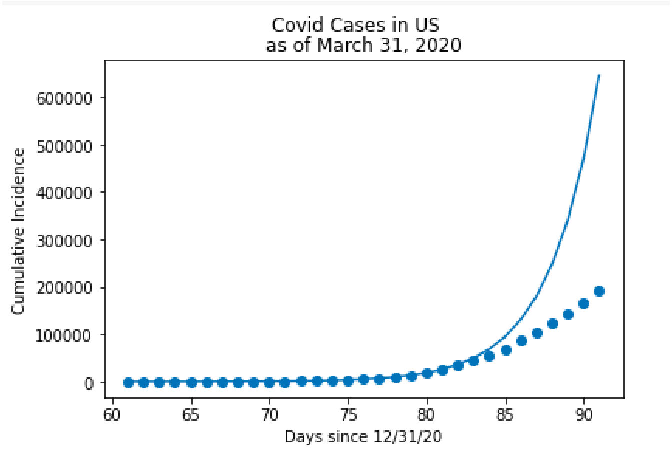
What was the percent increase in new covid cases per day?

Based on this equation, what would you predict the US cumulative incidence would be on March 31, 2020, if exponential growth continued?

In fact, US cumulative incidence on March 31, 2020 was 192,301.

Explain.

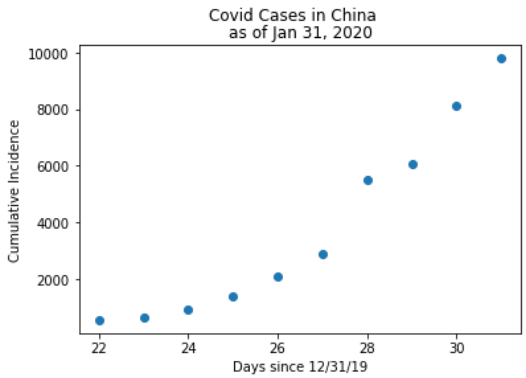




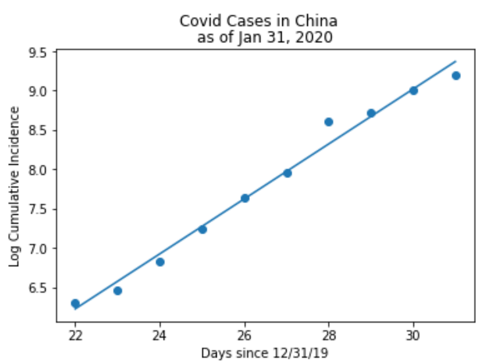
China

Let's perform the same sort of analysis for China.

Here is the cumulative incidence of cases in China through Jan 31.



Here is a log transformed plot with a fit line, with $m = 0.35$ and $b = -1.45$.



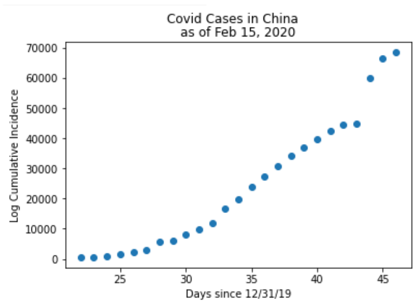
- What would be the best equation to fit to the original plot?
    1. $\text{cases} = 0.35 \cdot \text{time} - 1.45$
    2. $\text{cases} = \ln(0.35 \cdot \text{time} - 1.45)$
    3. $\text{cases} = e^{0.35 \cdot \text{time} - 1.45}$
    4. $\text{cases} = 0.23e^{0.35 \cdot \text{time}}$
    5. $\text{cases} = e^{0.35 \cdot \text{time}} + 0.23$

- By what percent was the cumulative number of covid cases growing by each day?

- Based on this equation, when do you think the first case in China happened?

- Based on this equation, what would you predict the cumulative incidence in China would be on Feb 15, 2020, if exponential growth continued?

- Compare to actual incidence in China on Feb 15, 2020, of 68,413.



- If exponential growth continued, when would you expect the cumulative incidence to reach 100,000,000?