Covid Vaccination and Wrap-Up

After completing this section, students should be able to:

- Define herd immunity
- Explain why an epidemic will start to die down when $r_e < 1$.
- Explain the relationship between the basic reproductive number of a disease and the level of vaccination needed to squash an outbreak.
- Use this relationship to calculate levels of vaccination needed.
Before Class

• Watch this 8.5 minute video on vaccination
• (Optional) read this article on herd immunity.
• Answer the questions:

1. What is the condition on the basic reproductive number that will prevent a disease from spreading in a susceptible population (i.e. the infection will die down before ever blowing up into an epidemic)?
   (a) If the basic reproductive number is less than 1, the disease won’t spread.
   (b) If the basic reproductive number is greater than 1, the disease won’t spread.
   (c) If the basic reproductive number is less than 2, the disease won’t spread.
   (d) If the basic reproductive number is greater than 1, the disease won’t spread.

2. Assume the basic reproductive number $r_0$ for coronavirus is 2.2 (which is the estimate from the W.H.O.) What percent of the population would need to be vaccinated to stop the spread of coronavirus in a susceptible population? (pick the closest answer)
   (a) About 75%
   (b) About 65%
(c) About 55%
(d) About 45%
Introduction

Let's see what happens if we add vaccination to our SIR model. Vaccinating people in advance of a disease outbreak affects which of these initial conditions? (Choose the best answer(s)) (PollEv)

A. $S(0)$
B. $I(0)$
C. $R(0)$

Which other parameters in our SIR model will vaccinating affect?

A. $r_0$
B. $r_e$
C. $a$
D. $b$
Vaccination in the SIR model

Return to the SIR the spreadsheet, and using a contagious period $k = 14$ days, $r_0 = 2.45$, $S(0) = 100000$, $I(0) = 100$, $R(0) = 0$, and the population size $N(0) = 100100$.

Make a cell that gives the vaccination level and set it at 0.8.

Now update $R(0)$ to account for this level of vaccination, without changing the population size, by using a formula. Using a formula means we can tweak this vaccination level easily and see the effects.

Update $S(0)$ also using a formula.

Does $I(0)$ need to be updated to keep the population size the same and keep the model making sense?

What do you notice from the graph about the epidemic?
Experiment with different levels of vaccination at time 0 by changing the vaccination rate, and \( S(0) \) and \( R(0) \) accordingly. What is the lowest level of vaccination that makes the number of infected people go strictly down, without ever going up?

Does this agree with the formula \( V = 1 - 1/r_0 \) from the video?

Try using a different value of \( r_0 \), and again, experiment with the level of vaccination until you have the smallest vaccination rate that makes the number of infected people go down, without going up. Check that it again agrees with the \( 1 - 1/r_0 \) formula.
The formula $1 - 1/r_0$

Let’s see if we can explain where the vaccination level formula $1 - 1/r_0$ comes from.

Why should an epidemic die down if $r_e < 1$?

Write down a formula relating $r_0$ and $r_e$.

Assuming you vaccinate at time 0, and no one is already immune from prior infections (e.g. a new disease), the vaccination level $V$ is related to the number of susceptible people at time 0 and the population size at time 0. Which of the following equations captures this relationship?

A. $V = \frac{S(0)}{N}$
B. $V = 1 - \frac{S(0)}{N}$
C. $V = 1 + \frac{S(0)}{N}$
D. $V \cdot N = S(0)$
Use this equation and the equation relating $r_e$ and $r_0$ to rewrite the inequality $r_e < 1$ in terms of $V$.

Solve for $V$ to get a formula that looks familiar

**Definition.** Herd immunity is ...

Use the formula $V = 1 - 1/r_0$ to see what vaccination level would be needed to establish herd immunity for a disease with $r_0 = 4.5$, in a population that has never been exposed to the disease.

Suppose we can only achieve a vaccination level of 65% for this disease, but can reduce $r_0$ by social distancing and other measures. What level would we have to reduce $r_0$ to, in order to achieve herd immunity?
Homework

1. Measles has a basic reproductive number between 12 and 18. Let’s take $r_0$ to be 15. Measles is contagious for about 8 days (about 4 days before rash appears to about 4 days after. Suppose you have a population of size of 100,000, with 100 infected people ($I(0) = 100$) and everyone else susceptible ($S(0) = 99900$, $R(0) = 0$), Put these numbers into the SIR model spreadsheet

(a) About how long does the measles epidemic last?

(b) About how many people total get the disease?

(c) On what day is $r_e < 1$?

2. Suppose you vaccinate 65% of the population, but use the rest of the parameters stay the same as the previous problem?

(a) About how long does the measles epidemic last?

(b) About how many people total get the disease? (remember, you will need to subtract the number of vaccinated people from the total number of removed people)

(c) On what day is $r_e < 1$?

3. (a) In theory, based on the formula $1 - \frac{1}{r_0}$, what fraction of the population do you
need to vaccinate to squash a measles epidemic?

(b) Verify this by putting that number into the SIR model spreadsheet and taking a screenshot of either the graph or the column of numbers of infectives.