Polar Coordinates

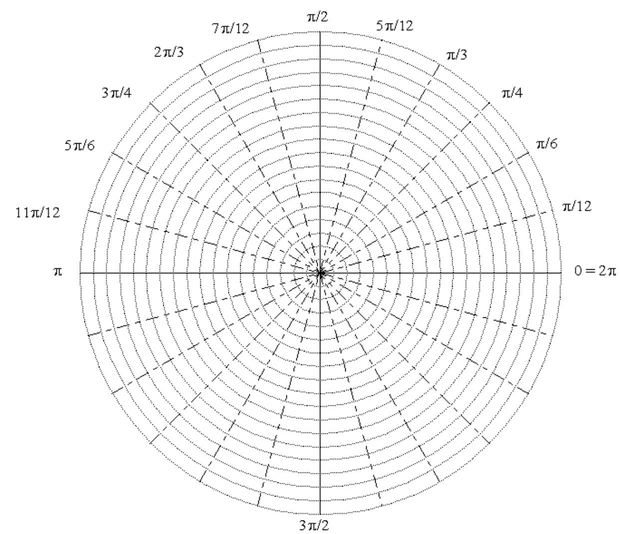
After completing this section, students should be able to:

- Graph a point given in polar coordinates.
- Explain how to graph a point given in polar coordinates when the given r-value is negative.
- Find more than one way to represent a point in polar coordinates.
- Convert points from polar coordinates to rectangular coordinates and vice versa.
- Convert equations from rectangular coordinates to polar coordinates.
- Graph simple equations in polar coordinates, such as $\theta = 2$, $r = 3$, $r \sin\theta = 5$, $r = cos\theta$.
- Match graphs and equations in polar coordinates.
- Predict the symmetry of a graph from its equation in polar coordinates.

Cartesian coordinates: (x, y)
Polar coordinates: (r, θ), where r is:
. and θ is:

Example. Plot the points, given in polar coordinates.

1. $(8, -\frac{2\pi}{3})$



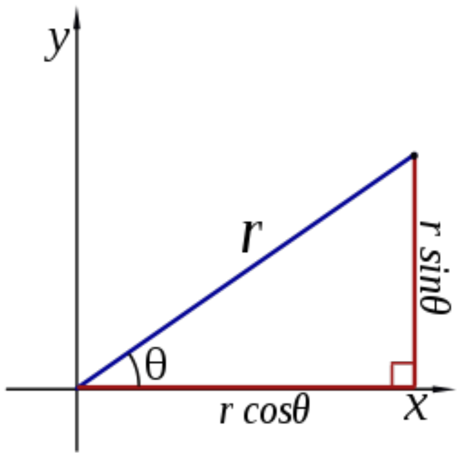
2. $(5, 3\pi)$

3. $(-12, \frac{\pi}{4})$

Note. A negative angle means to go clockwise from the positive x-axis. A negative radius means jump to the other side of the origin, that is, $(-r, \theta)$ means the same point as $(r, \theta + \pi)$

Note. To convert between polar and Cartesian coordinates, note that:

- $x =$ ________
- $y =$ ________
- $r =$ ________
- $\tan\theta =$ ________



Example. Convert $(5, -\frac{\pi}{6})$ from polar to Cartesian coordinates.

Example. Convert $(-1, -1)$ from Cartesian to polar coordinates.

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Review. Points on the plane can be written in terms of rectangular coordinates (a.k.a. Cartesian coordinates) (x, y) or in terms of polar coordinates (r, θ)
where r represents ...

and θ represents ...

The quantities x and y and r and θ are related by the equations ...

Review. Find the rectangular coordinates of a point with the polar coordinates $\left(5, \frac{7\pi}{4}\right)$

A. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

B. $\left(\frac{-\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

C. $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$

D. $\left(\frac{-5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

Review. Find the polar coordinates of a point with rectangular coordinates $\left(-2, -2\sqrt{3}\right)$. More than one answer may be correct.

A. $\left(1, \frac{\pi}{3}\right)$

B. $\left(4, \frac{\pi}{3}\right)$

C. $\left(4, \frac{4\pi}{3}\right)$

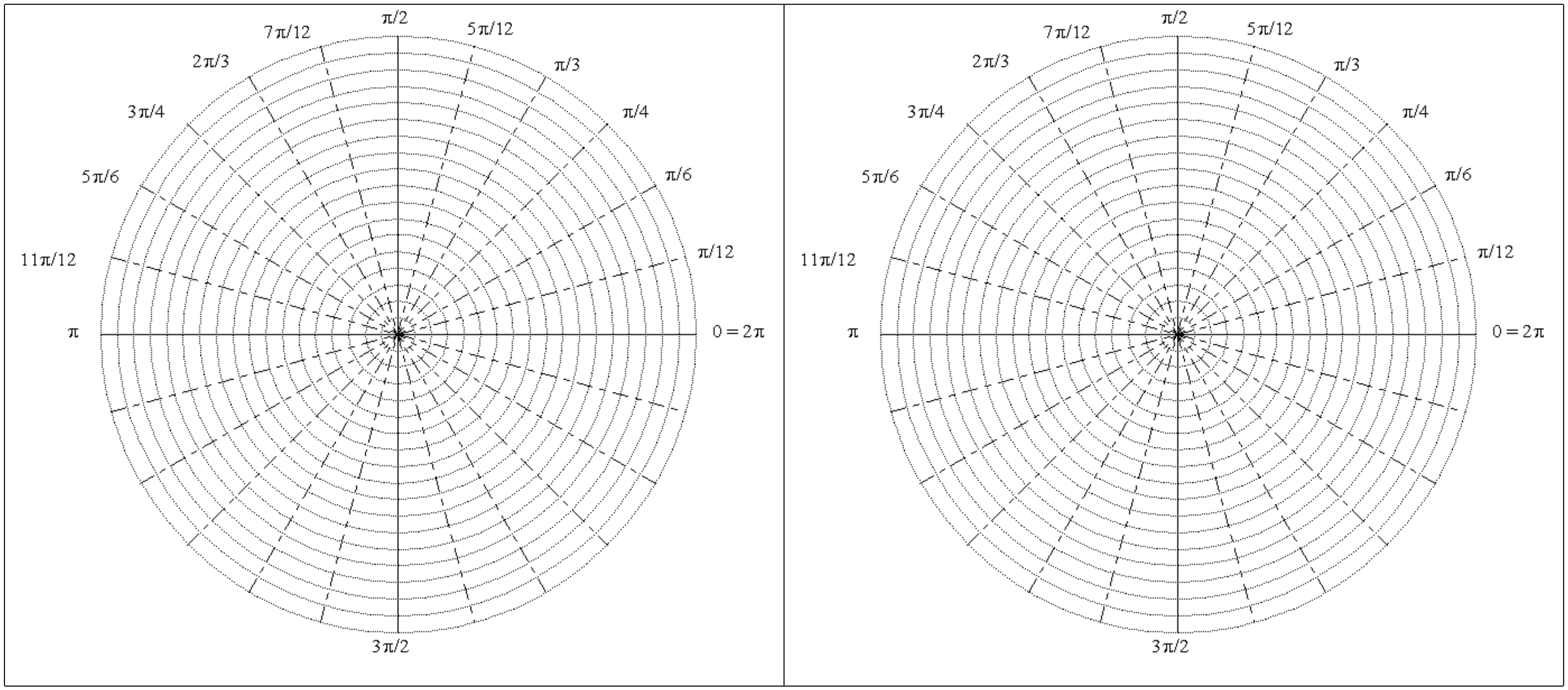
D. $\left(4, -\frac{2\pi}{3}\right)$

E. $\left(-4, \frac{\pi}{3}\right)$

Example. Plot the following curves and rewrite using Cartesian coordinates.

1. $r = 7$

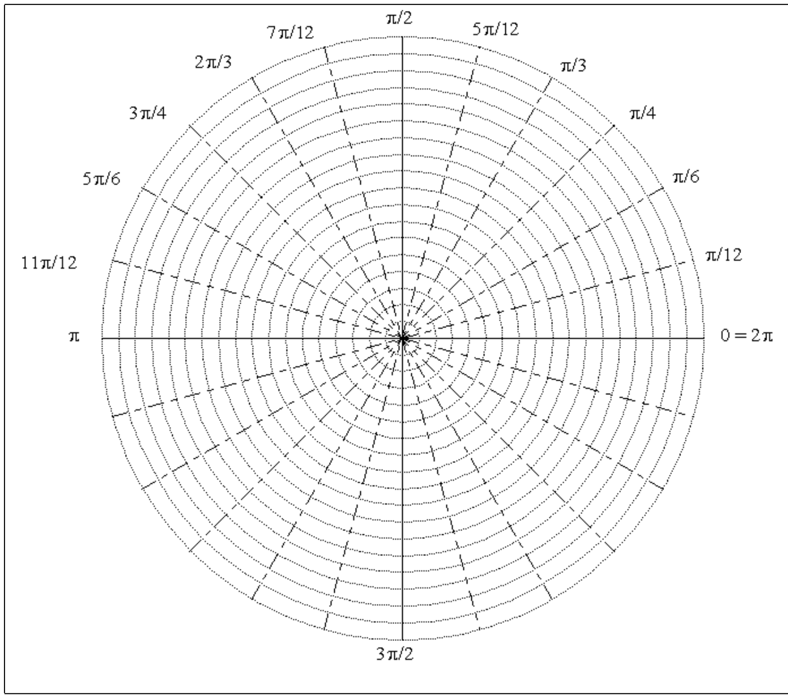
2. $\theta = \dfrac{\pi}{3}$

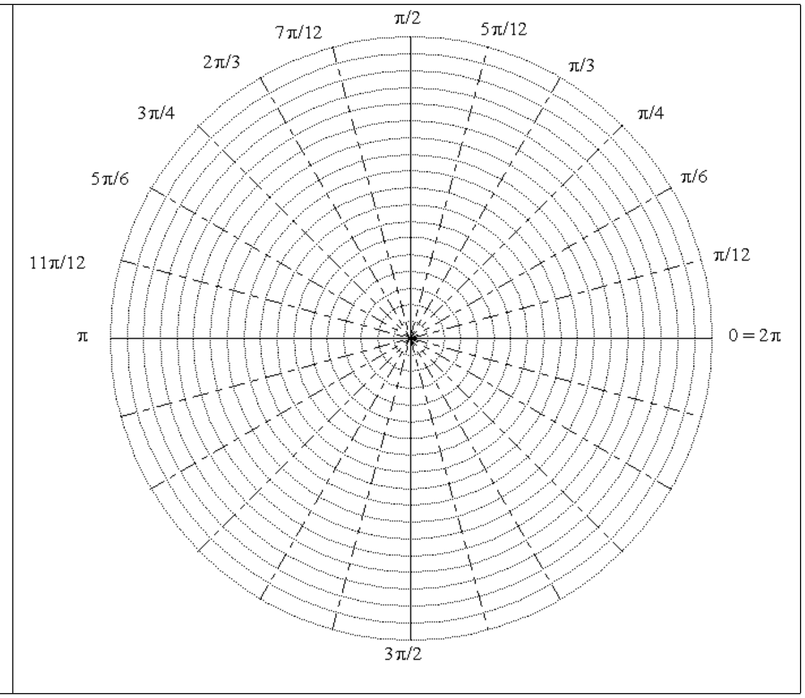


Example. Plot the following curves and rewrite the first one using Cartesian coordinates.

3. $r = 12\cos(\theta)$

4. $r = 6 + 6\cos(\theta)$ (an example of a limacon)





Example. Convert the Cartesian equations to polar coordinates:

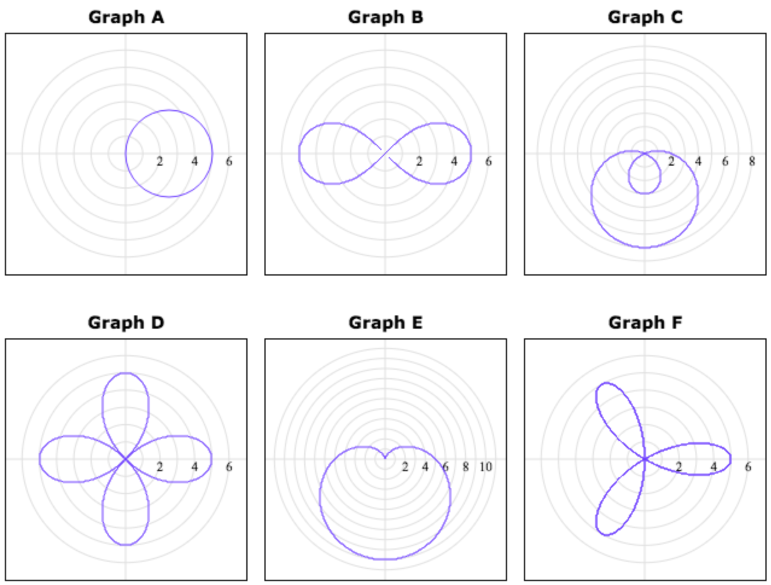
1. $y = -2$

2. $y = x$

3. $y^2 + (x - 5)^2 = 25$

4. $4y^2 = x$

Example. Match the polar equations with the graphs. See how many you can guess without using graphing software, then use graphing software to help.



1. $r = 5\cos\theta$
2. $r = 5 - 5\sin\theta$
3. $r = 5 - 8\sin\theta$
4. $r = 5\cos 2\theta$
5. $r = 5\cos 3\theta$
6. $r^2 = 5\cos 2\theta$

Parametric Equations

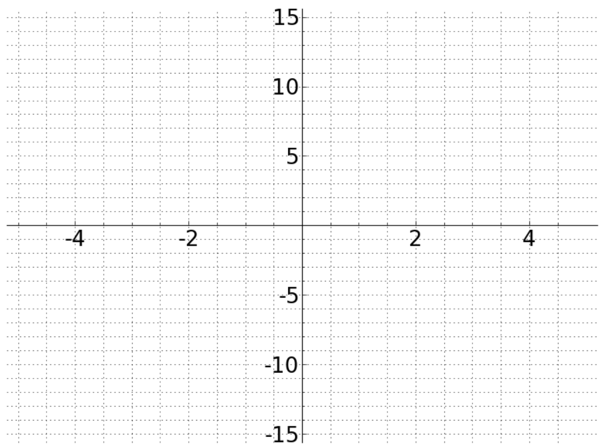
After completing this section, students should be able to:

- Draw the graph for a curve described with parametric equations by plotting x and y values corresponding to various t values.
- Convert from parametric equations to a rectangular (Cartesian) equation.
- Convert from a Cartesian equation to parametric equations.
- Write the equation for a circle or ellipse in parametric equations.
- Write the equation for a line in parametric equations.

Definition. A **cartesian equation** for a curve is an equation in terms of x and y only.

Definition. Parametric equations for a curve give both x and y as functions of a third variable (usually t). The third variable is called the **parameter**.

Example. Graph $x = 1 - 2t$, $y = t^2 + 4$



t	x	y
-2	5	8
-1	3	5
0		

Find a Cartesian equation for this curve.

Example. Plot each curve and find a Cartesian equation:

1. $x = \cos(t)$, $y = \sin(t)$, for $0 \leq t \leq 2\pi$
2. $x = \cos(-2t)$, $y = \sin(-2t)$, for $0 \leq t \leq 2\pi$
3. $x = \cos^2(t)$, $y = \cos(t)$

Example. Write the following in parametric equations:

1. $y = \sqrt{x^2 - x}$ for $x \le 0$ and $x \ge 1$

2. $25x^2 + 36y^2 = 900$

Example. Describe a circle with radius r and center (h, k):

a) with a Cartesian equation

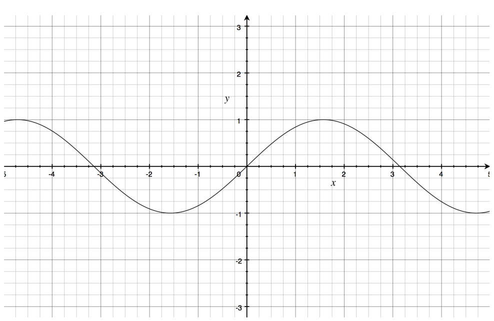
b) with parametric equations

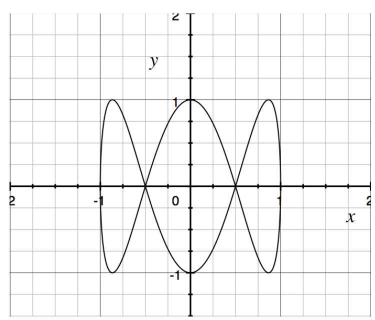
END OF VIDEO

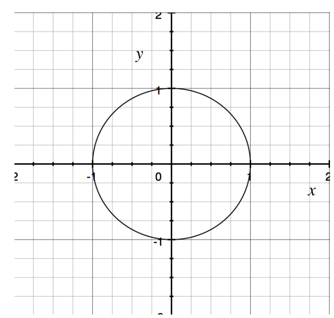
Review. Cartesian equations are ...

Parametric equations ...

Review. Which of the following graphs represents the graph of the parametric equations $x = \cos t$, $y = \sin t$. (The horizontal axis is the x-axis and the vertical axis is the y-axis.)

A. 

B. 

C. 

Example. Find a Cartesian equation for the curve and graph the curve with an arrow to represent direction of motion.

Methods:

1. $x = 3 + 2t, y = -5 - 4t, -2 \leq t \leq 2$

2. $x = 1 + 3\cos(t) + 3, y = 1 + 4\sin(t)$

3. $x = e^{2t}, y = e^{-4t}$

Example. Find parametric equations for the curve.

Methods:

1. $x = -y^2 - 6y - 9$

2. $\dfrac{x^2}{9} + \dfrac{y^2}{49} = 1$

3. $4(x-2)^2 + 25(y+1)^2 = 100$

Extra Example. What is the equation for a circle of radius 8 centered at the point (5, -2)

1. in Cartesian coordinates ?

2. in parametric equations?

Extra Example. Find parametric equations for a line through the points $(2, 5)$ and $(6, 8)$.

1. any way you want.

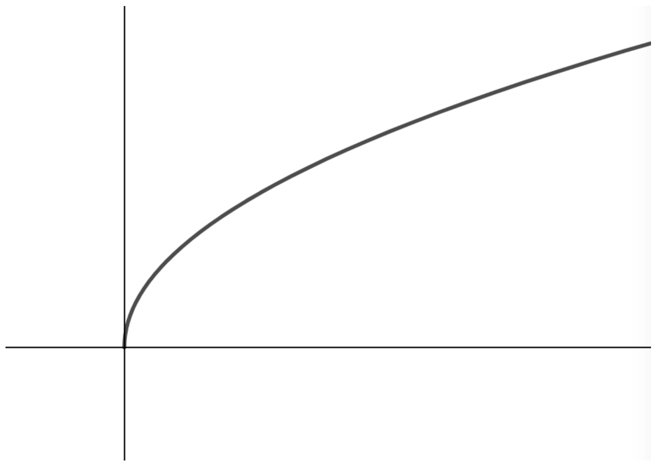
2. so that the line is at $(2, 5)$ when $t = 0$ and at $(6, 8)$ when $t = 1$.

Difference Quotient

After completing this section, students should be able to:

- Explain what a difference quotient represents in terms of a graph of a function or in a context like when the function represents distance travelled over time.
- Calculate and simplify the difference quotient of a various functions, including linear, quadratic, rational, and square root functions.

For a function $y = f(x)$,

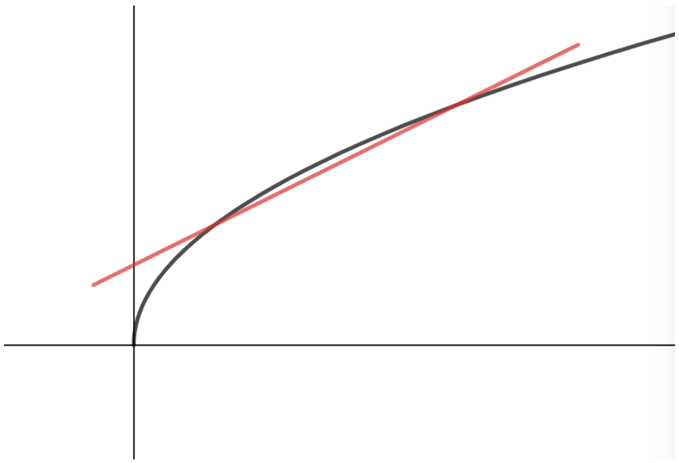


Definition. A secant line is

Definition. The average rate of change for $f(x)$ on the interval $[a, b]$ is

Example. The average rate of change for $f(x) = \sqrt{x}$ on the interval $[1, 4]$ is

Definition. A difference quotient represents



Example. Find and simplify the difference quotient for $f(x) = 2x^2 - x + 3$

Review. Which of the following statements are true?

A. The average rate of change of a function $f(x)$ on the interval $[a, b]$ is given by the formula $\dfrac{f(b) - f(a)}{b - a}$.

B. The average rate of change of a function represents the slope of a secant line.

C. The difference quotient represents an average rate of change.

D. The difference quotient is given by the formula $\dfrac{f(x) - f(h)}{x - h}$

Example. Find the average rate of change of $f(x) = 2x^2 - 2x + 3$ from $x = -2$ to $x = 1$.

Example. Find the difference quotient for $f(x) = 2x^2 - 2x + 3$

Example. Find and simplify the difference quotient for $f(x) = \dfrac{5}{x-6}$

Extra Example. Find and simplify the difference quotient for $f(x) = \sqrt{x+4}$.