§11 Kaleidoscopes

The goal for this lesson is to experiment with different mirror arrangements and the resulting symmetry patterns.

Reference:

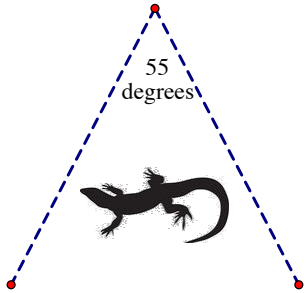
- Symmetry, Shape, and Space by Kinsey and Moore.

Supplies:

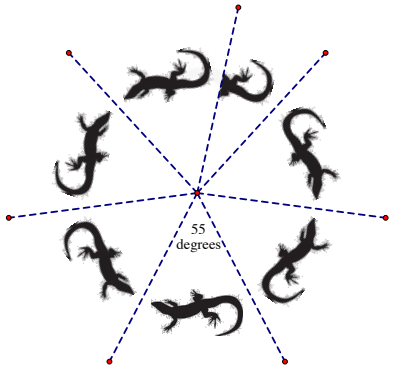
- mirrors
- caps
- paper rolls
- beads and rubber bands other decorative items
- tape
- paper
- felt
- plastic transparencies
- scissors
- eyepieces

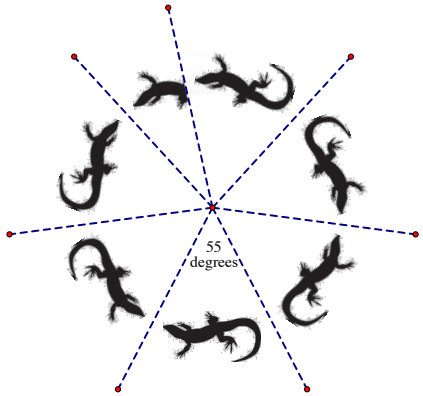
Hinged Mirrors

You can make a hinged mirror out of two mirrors and tape, or use one of the hinged mirror kits. The mirrors should be upright and perpendicular to the table.

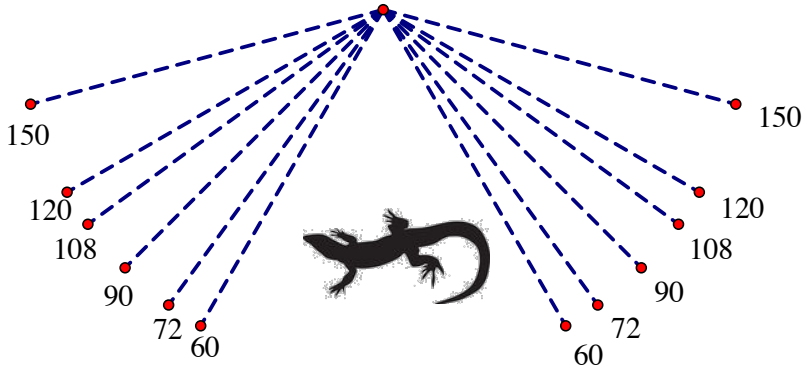


Place your hinged mirror on the lizard picture above, with the hinge at the dot and the mirrors lined up on the dotted lines. It is possible to see both of the images below. Describe how this can be done.





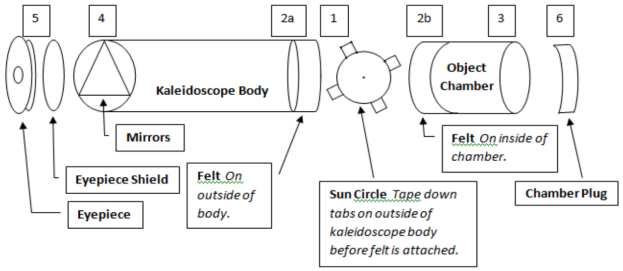
Below is a lizard with lines forming various angles. For each angle, place your hinged mirrors so that the edges lie along the relevant dotted lines and the hinge is at the dot. Describe what you see.



Angle measure measure	Number of images including the original	# right handed lizards	# left handed lizards
30°			
45°			
60°			
72°			
108°			
120°			
150°			
n°			

Give a general formula for the relationship between the angle and the number of lizards. When is there an equal number of lizards of each orientation?

You will need a cardboard tube or PVC pipe about 9 inches long for the kaleidoscope body, a shorter tube of slightly larger diameter for the object chamber, 3 mirrors, felt strips,a plastic "sun circle", beads or other decorative objects, and a plastic plug.



1. If necessary, cut the tube for the kaleidoscope body so that it is between 8.5 and 9 inches long.
2. (Optional) Make a transparent plastic "sun circle" and tape it down onto end of kaleidoscope body. This piece will keep beads in the object chamber from falling into the main body of the kaleidoscope.
3. Attach 1/2 inch wide felt strips to the outside of one edge of the kaleidoscope body and the inside of one edge of the object chamber.
4. Slide the object chamber over the kaleidoscope body so that the felt strips line up and meet. The object chamber should extend past the body.
5. Form an equilateral triangle with three mirrors and push it into the kaleidoscope body. Remove the protective film before inserting the mirrors.
6. Add beads to the object chamber plug until about half of the plug is covered. Insert the plug into the object chamber.
7. (Optional) Insert a white plastic eyepiece.
8. (Optional) Cut paper to fit around the kaleidoscope body and decorate.

Mutant kaleidoscopes

- The standard kaleidoscope has three mirrors that form an equilateral triangle.
- Experiment with using mirrors that form a triangle that is not equilateral, or a different polygon like a square.
- What configurations make whole numbers of shapes?

Problems on kaleidoscopes

1. We saw that hinged mirrors at an angle of $\theta = \frac{n}{360}$ will make n copies of the lizard, and if n is an odd number, then there will be a different number of left-tailed lizards than right-tailed lizards. Assuming that your original lizard is left-tailed, which odd numbers n will give you more left-tailed lizards than right-tailed lizards, and which will give you more right-tailed lizards than left-tailed lizards?
2. Take a photo of your "kaleidoscope image": that is, what you see when you look through your kaleidoscope and post it on the Kaleidoscope Padlet.
3. It's no surprise that your kaleidoscope image has reflection symmetry since it is made of mirrors.
   (a) Does it also have (approximate) rotation symmetry? If so, through what rotation angle(s)?
   (b) Does it also have (approximate) translation symmetry? If so, how long are the translation vectors and in what direction? (Your answer should be in terms of the length and position of the mirrors.)
   (c) Does it also have (approximate) glide symmetry? If so, where are the glide lines and how long are the glide translation vectors?

(Remember, a "symmetry" is an isometry that leaves the figure unchanged. None of the symmetries for our images will be exactly perfect, since we made the kaleidoscopes out of cardboard and plastic mirrors, so just look for approximate symmetries here.)