§13 Finite figures - optional

The goal for this part is relate “rosettes” and “finite figures”.
Rosettes are examples of finite figures.
What is finite about them?
Definition: a \textit{finite figure} is a figure that has only finitely many symmetries; that is, finitely many isometries that preserve the figure. Are there any finite figures that are NOT rosettes (that is, not of the type $C_n$ or $D_n$)?
Is it possible to have a finite figure (that is finitely many isometries) with translation as one of the isometries?

Is it possible to have a finite figure (that is finitely many isometries) with glide as one of the isometries?
What arrangements of mirrors are possible? Is it possible to have two parallel mirror lines?

What arrangements of rotations are possible? Is it possible to have two rotations through the same angle, but with different rotocenters?
Fill in explanations for each star in the flow chart.
Start with a figure with finitely many symmetries

- Are there translations?
  - YES: There are translations
  - NO: Are there glide reflections?
    - YES: There are glide reflections
    - NO: Are there rotations?
      - YES: There are rotations
      - NO: Are there reflections?
        - YES: There are reflections
        - NO: C1

- Contradiction! There are actually infinitely many isometries!

- Are there at least two distinct rotocenters, each with the same angle of rotation?
  - YES: There is one unique rotocenter
  - NO: Are there any parallel mirror lines?
    - YES: Are there any intersecting mirror lines?
      - YES: D1
      - NO: G
    - NO: D1
Which of the following statements are true?

1. If a figure is a rosette, then it has finitely many symmetries.
2. If a figure has finitely many symmetries, then it is a rosette.
3. If a figure is a rosette, then it has no translation symmetries.
4. If a figure has no translation symmetries, then it is a rosette.
5. If a figure has finitely many symmetries, then it has no translation symmetries.
6. If a figure has no translations symmetries, then it has finitely many symmetries.