§14 Frieze Patterns

The goal for this part is to identify the symmetries in frieze patterns.

Reference:

- Symmetries of Things, Chapter 5, pp. 66 - 73

Supplies:

- Paper strips for paper dolls, scissors, tape.

What are frieze patterns?

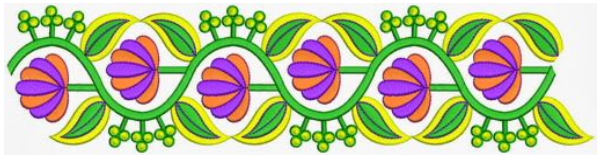
A *frieze pattern* is a pattern that has translation symmetries in (only) one direction.

They are also called *strip patterns* and *border patterns*.

We will usually assume our strip pattern is oriented so that translations are in the left right direction. That is, the center line of the strip is horizontal.

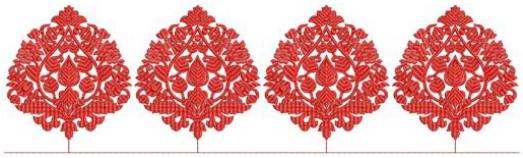
Which of these are frieze patterns?

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

Symmetries of frieze patterns

For each of the strip patterns, mark all distinct symmetries as follows:

1. Mark the smallest translation in the left to right direction with an arrow.
2. Draw mirror lines.
3. Mark rotocenters with circles.
4. Mark the glide lines and mark the (smallest) translation vectors for glide reflections with arrows.

Fundamental domain of frieze patterns

- For any frieze pattern, a **fundamental domain** is a piece of the pattern of the smallest possible area that can be repeated to make the entire pattern, *using isometries that are symmetries of the entire pattern*.
- For each of the frieze patterns above, identify a possible fundamental domain and mark it.
- (Suggestion: as a first step, look for a translational symmetry and reduce from there.)

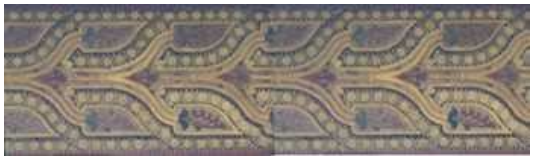
Practice problems

Ignore minor color differences and other minor asymmetries.

A. 

B. 

C. 

D. 

E. 

F. 

Mosaic
Nuestra Senora de la Almundena
Madrid, Spain

Tile Frieze
Palacio de Velazquez
Parque de Retiro
Madrid, Spain

Back of a Bench
Banos de la Maria de Padilla Reales Alcazares
Seville, Spain

Ceiling
Mezquita
Cordoba, Spain

Mosaic Border
Alcazar de los Reyes Cristianos
Cordoba, Spain

Meander Frieze
San Giorgio Maggiore
Venice, Italy

From the Mathematical Association of America.

1. For each pattern in the collection of European frieze patterns, do the following:
   (a) Mark a fundamental domain by drawing a line around it.
   (b) List all the types of symmetries that it has, using the following letters: H for reflection through a horizontal mirror line, V for reflection through a vertical mirror line, R for rotation, G for glide reflection, and T for translation.
2. Draw three different frieze patterns, either using Geogebra or on paper. Try to make your examples different from each other by using different motifs AND by incorporating different types of symmetry.