Project ideas

- Work in groups of 1, 2, 3, or 4
- You can choose your own group and your own topic. Topic suggestions are below, but you are not limited to this list. Please pick a topic and a group by the end of class on Monday.
- You will have about 5 hours of class time to work on your project and put together a 15 minute presentation (e.g. on Google Slides)
- Include references for your sources of information in any format.
- Your group will present your project on Friday morning.

Proof projects from class notes that we skipped

1. Analyze the game of criss-cross for triangles and other shapes. Is there a winning strategy? Prove it! See the class notes that we skipped.
2. Explain the definition of regional curvature in terms of the celestial sphere and use it to show that the two other definitions of regional curvature that we used in class are actually the same (the orange peel splitting definition and the pointwise curvature times the area definition). See the class notes that we skipped.

3. Prove that the only isometries in 2-d are translations, reflections, rotations, and glides (if not done in class). See the class notes that we skipped.
4. Prove that the only "finite figures" (figures with a finite symmetry group) are the C_n and D_n patterns. See the class notes that we skipped.

Maker Projects

5. Hexaflexagons - make them and analyze them. Hexaflexaflakes (see Vi Hart Video)
6. Sphereflakes. Make them and analyze them. See Vi Hart Video and plastic beach ball supplies.
7. Pick a musical motif and write a short composition using one of the frieze symmetry patterns. You can use create a free Noteflight account at www.noteflight.com to help. Use the mouse to place notes on lines or space. Use the [and] keys and the period to change to different types of notes, or use the floating toolbox of symbols. Use the up and down arrows to move notes or whole measures up or down. Use control-Z to undo, and Edit ¿ Cut and Paste and Repeat to repeat things. Alternatively, you can use Musescore.org and Musescore.com. Or, use the app flat.io. See also the music boxes and score paper in the classroom supplies.
8. Use the 3-d printer in the library to make models of the projective plane, including the cross-cap model and boys surface model, and / or other topological surfaces or

geometric objects.

9. Escher style patterns. Draw Esher-style interlocking patterns with representable images for a sample of symmetry types.
10. Draw Escher style wallpaper patterns using Conway's criteria.
11. Colorings of tilings. For a few of your favorite Escher tessellations (or homemade tessellations, or other tessellations), demonstrate all possible ways of coloring the pattern in a symmetric way. See Chapters 11 and 12 of The Symmetries of Things for tables of 2-fold and 3-fold colorings.
12. Make a torus out of paper See this article.
13. Spherical symmetry in origami. Build paper models of as many different types of spherical symmetry as possible. Check out http://www.cutoutfoldup.com/ for some ideas. See also George Hart's constructions slide together models.
14. Tiling patterns with origami. Build origami patterns with frieze or wallpaper symmetry. Or do the same thing using knitting.
15. Spherical symmetry Temari balls. Embroider Temari balls or make models of spherical symmetry with patches of cloth covering foam balls, or use another method to make models of spherical symmetry.
16. Spherical symmetry kaleidoscopes. Kaleidoscopes with tapered sides make truly amazing spherical symmetry patterns. Here is a sample some more samples and

some videos (the Quilt Kaleidescope video has a picture of the set-up) . Page 63 of The Symmetries of Things has some guidelines for building them.

17. Three-dimensional pattern kaleidoscopes. Symmetry, Shape, and Space: An Introduction to Mathematics Through Geometry by Kinsey and Moore has information on a different kind of kaleidescope, that puts mirrors together in the pattern of a cube and other patterns to make infinite repeating figures in 3-dimensions.

18. Hyperbolic quilt. Sew a hyperbolic patchwork quilt. Or, devise your own pattern. Or knit or crochet one.

19. Hyperbolic tilings on the Poincare disk. It is possible to use Geogebra or other software to draw hyperbolic tilings similar to M. C. Escher's circle limit series and other artwork.

Math Research Projects

20. Map coloring. Study map coloring on the sphere and other surfaces. Prove that you only need at most 5 colors to color a map on the sphere so that adjacent countries always have different colors. Prove that you could need up to seven colors for a map on the torus, but no more. How many colors are needed for other surfaces? See Sue Goodman's Beginning Topology book.

21. Read more of the Shape of Space and present the 8 geometries on 3-manifolds.

22. Three dimensional isometries. In the 2-dimensional plane, we only needed four types of motions to describe all isometries: reflection, rotation, translation, and glide reflection. Are all these motions possible in 3-dimenstions? Are their other isometries possible in 3-d? What happens when you combine isometries in 3-d? Is a reflection of a reflection still always a rotation or translation? Etc. What about isometries in 4-dimensions?!

23. Groups. In math, a group is a bunch of things (the "group") and a way of combining pairs of things. The symmetries of a finite figure can be described as a group, because when you combine two symmetries by following one by the other you get another symmetry of the finite figure. See sections 3.2 through 3.4 and chapter 6 of Groups And Symmetry: A Guide to Discovering Mathematics by David Farmer. Wallpaper patterns also have group structures. Explain groups and analyze the symmetry groups of a few wallpaper patterns.

24. Seifert Surfaces. How can you build a surface that a knot bounds? Can you always build an orientable surface that the knot bounds? How? What does the genus of the surface tell you about the knot? Note: you can build these spanning surfaces, called Seifert surfaces, out of wire and soap bubble solution. See Beginning Topology by Sue Goodman, Section 7.7.

25. Map Projection. Since the earth is curved, there is no way to make a flat map without cutting gaps or stretching and distorting some parts. Research the types

of maps that can be made to represent the earth and the pros and cons of each. The same thing can be done for hyperbolic plane, instead.

26. Hyperbolic plane models. Compare and contrast different models of the hyperbolic plane. What are the advantages and disadvantages of each?

27. Double Strip Patterns. Report on the types of double strip patterns analyzed in this paper and convert the signatures to orbifold notation.

28. Crystallographic patterns (three dimensional wallpaper patterns). See The Symmetry of Things, Chapters 22 and 25. How many are there? Do they all actually occur in real crystals? What notatation can be used to describe and categorize them? Do they have other appearances in art and nature besides rock crystals?

Applications Projects

29. Knot Applications. Explore the applications of knot theory to biology, chemistry, and physics. See The Knot Book by Colin Adams, Chapter 7.

30. Soccer Ball History. Research the history of soccer ball patterns. When did the standard black and white pattern become popular? What are the all the official world cup patterns and what symmetry types do they have? Are there any symmetry types that are not used for World Cup balls, and if so, which and why might they not be used? Etc.

31. Study wallpaper or frieze symmetry patterns used in a particular style of art., for example, 19th century American quilting patterns, or Islamic lattice patterns, or Chinese pottery from a particular era, or Native American basket weaving. Are all possible symmetry patterns represented? If not, can you come up with a theory for why certain patterns are absent? Are some patterns more common than others? Why? (See section 5.5 in Symmetry, Shape, and Space book by Kinsey and Moore for Islamic lattice patterns.
32. Escher. Analyze the wallpaper patterns used in Escher's tessellating patterns. Does he use all 17? Does he use some types more frequently during different periods of his life?
33. Analyze frieze patterns used in architecture. Are all 7 types commonly used? Are some types used more in some types of architecture?
34. Frieze patterns in clothing. Are all 7 types equally represented?
35. Wallpaper patterns in clothing. Are some pattern types used more in men's clothes (e.g. neck ties) and other types used more in women's clothes (e.g. dresses or scarves)?
36. Frieze patterns in music. What types of frieze patterns occur in what styles of music? Crab Cannon by Bach is one famous example. See this article by Vi Hart and this article by Alissa Crans et al. for more ideas.

37. Symmetry in biology. Which types of organisms have finite symmetry patterns and which don't? What types of organisms show frieze pattern symmetry? Wallpaper symmetry? Fractal symmetry?

38. Study applications of fractals.